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Variational Analysis and Sequential Quadratic Programming Approach for Robotics

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Abstract

Sequential quadratic programming (SQP) methods are widely used for solving practical optimization problems, especially in contact mechanics. The general structure of SQP methods is briefly introduced and it is shown how these methods can be adapted to field of Robotics, especially bipedal robot. Numerical results are presented for compass bipedal robot. This paper describes relationship between variational analysis and sequential quadratic programming using discrete mechanics and optimal control for bipedal robot.

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1. Introduction

Variational arguments are classical techniques whose use can be traced back to the early development of the calculus of variations and further. Rooted in the physical principle of least action they have wide applications in diverse fields. The discovery of modern variational principles and nonsmooth analysis further expand the range of applications of these techniques.Variational principles play a fundamental role in physics and mechanics.They state that a system adjusts its state always in a manner such that the associated functional is extremal.One can determine equilibrium states in a canonical way by studying local or global minimizers or maximizers of a functional [2]. Applications are found in many areas include:(a) linear and nonlinear elastostatics and contact problems (b) free boundary problems and multiphase problems (c) equilibria in reaction-diffusion systems (d) ground states in quantum mechanics and density functional theory (e) optimization problems (f) risk minimization in finance and economy [7]

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2. Variational Principle in Dynamic Behaviour

A dynamical systems with the property that their *evolution* satisfies certain optimality; so not just one state, but the whole evolution between certain specified initial and final times. We treat two classes, Lagrangian and Hamiltonian systems.

2.1. Lagrangian Systems

Let Q be the *configuration space* of a dynamical system. For discrete systems, Q will be a subset of $\mathcal{R}^N$, denoting the set of (generalized) coordinates that describe the position in space of the system. For continuous systems, Q will be some (subset of a) function space.

If we denote a particular state by $u(t) \in Q$, and the evolution as a trajectory $t \mapsto u(t) \in Q$, the velocity can be interpreted as an element form the tangent space [3]:

$$\partial_t u \in T_u Q.$$

A *Lagrangian* is a function(al) defined on the tangent space:

$$L : \mathcal{R} \times Q \times T_Q \in \mathcal{R}, \quad L = L(t, u, v),$$

with the aid of which a so-called *action functional* can be defined: for evolutions $t \mapsto u(t)$ with $t \in [t_0, t_1]$

$$\mathcal{A}(u) = \int_{t_0}^{t_1} L(t, u(t), \partial_t u(t))dt$$

Definition 2.1. *A dynamical system is called a Lagrangian system if a Lagrangian can be defined as above such that the actual evolutions of the system are critical points of the corresponding action functional.*

2.2. Hamiltonian Systems

Hamiltonian systems are systems that can also be found from a variational principle: *the canonical action principle.*

With $q \in Q$ (positive) and p (momentum) as variables, the state of the system is described by the pair (q, p); this is often called the *phase space*. A *Hamiltonian* is a function(al) on the cotangent space:

$$H = H(t, q, p)$$

A so-called *canonical action functional* is defined for evolutions $t \mapsto (q(t), p(t))$ with $t \in [t_0, t_1]$

$$\mathcal{A}_c(q, p) = \int_{t_0}^{t_1} [< p(t), \partial_t q(t) > - H(t, q(t), p(t))]dt.$$

Definition 2.2. *A dynamical system is called a Hamiltonian system if a Hamiltonian can be defined as above such that the actual evolutions of the system are critical points of the corresponding canonical action functional.*

Observation: In many problems form classical and continuous mechanics, *the Hamiltonian is the sum of kinetic and potential energy, i.e. the total energy,* both expressed in terms of the canonical variables from the phase space.

Hamilton's equations for a system with Hamiltonian H are the Euler-Lagrange equations of the canonical action functional; the are readily found to be [3]

$$\partial_t q = \frac{\partial H}{\partial p}$$
$$\partial_t p = -\frac{\partial H}{\partial q}$$

3. Structure of SQP

Sequential Quadratic Programming (SQP) is one of the most successful methods for the numerical solution of constrained nonlinear optimization problems. It relies on a profound theoretical foundation and provides powerful algorithmic tools for the solution of large-scale technologically relevant problems.

We consider the application of the SQP methodology to nonlinear optimization problems (NLP) of the form

$$\text{minimize } f(x)$$
$$\text{over } x \in \mathbb{R}^n$$
$$\text{subject to } h(x) = 0 \text{ and } g(x) \leq 0,$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is the objective functional, the functions $h : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^n \to \mathbb{R}^p$ describes the equality and inequality constraints [4]. The NLP shown above contains as special cases linear and quadratic programming problems, when f is linear or quadratic and the constraint functions h and g are affine.

SQP is an iterative procedure which models the NLP for a given iterate x^k, $k \in \mathbb{N}_0$, by a Quadratic Programming (QP) subproblem, solves that QP subproblem, and then uses the solution to construct a new iterate x^{k+1}. This construction is done in such a way that the sequence $(x^k)_{k\in\mathbb{N}_0}$ converges to a local minimum x^* of the given NLP as $k \to \infty$. In this sense, the NLP resembles the Newton and quasi-Newton methods for the numerical solution of nonlinear algebraic systems of equations. However, the presence of constraints renders both the analysis and the implementation of SQP methods much more complicated.

4. Dynamic Modeling

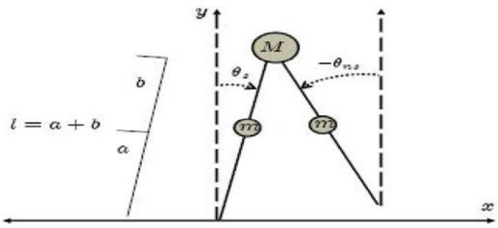

Fig. 1. Model of Compass Gait Biped

The application of discrete variational principles allows for the construction of an optimization algorithm that enables the discrete solution to inherit characteristic structural properties from continues problem. The DMOC optimization problem formulated using boundary conditions is solved by SQP methods for the bipedal robot.

4.1. Lagrangian Dynamics

As in [5], consider a mechanical system with a configuration space, Q, assumed to be a smooth manifold with a tangent bundle, TQ. The mechanical systems we will take into consideration have Lagrangians, $L : TQ \to \mathbb{R}$, given in coordinates by:

$$L(q, \dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} - V(q),$$

Applying Hamiltonian's Variational principle to these systems yields Euler–Lagrange equations of the form:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q) = 0,$$

In controlled cases, using the Lagrange–D'Alembert Principle will yield equations of motion of the form:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q) = Bu,$$

4.2. Discrete Mechanics and Optimal Control Approach for Bipedal Robot

the equations of motion for a forced system with Lagrangian dynamics, such as a SHMS, follow from the *Lagrange–d'Alembert principle.*

The principle requires that

$$\delta \int_0^T L(q(t), \dot{q}(t))dt + \int_0^T u(t) \cdot \delta q(t)dt = 0$$

for all variations δq with $\delta q(0)\delta q(T) = 0$. The work in [6] sets up optimal control problems as constrained nonlinear optimization problems by utilizing a discretization of this variational principle. The method begins by discretizing the trajectory $q(t)$ in the same manner as in variational integrator theory [1]. A discrete version of the Lagrange–d'Alembert principle.

$$\delta \sum_{k=0}^{N-1} L_d(q_k, q_{k+1}) + \sum_{k=0}^{N-1} u_k^- \cdot \delta q_k + u_k^+ \cdot \delta q_{k+1} = 0,$$

for all variations $\{\delta q_k\}_{k=0}^N$ with $\delta q_0 = \delta q_N = 0$. This is equivalent to the system of *forced discrete Euler–Lagrange equations*

$$D_2 L_d(q_{k-1}, q_k) + D_1 L_d(q_k, q_{k+1}) + u_{k-1}^+ + u_k^- = 0,$$

Standard and discrete Legendre transforms yields the discrete boundary conditions

$$D_2 L(q(0), \dot{q}(0)) + D_1 L_d(q_0, q_1) + u_0^- = 0,$$
$$-D_2 L(q(1), \dot{q}(1)) + D_1 L_d(q_{N-1}, q_N) + u_{N-1}^+ = 0.$$

The final step is to note that the continuous time cost functional

$$J(q, u) = \int_0^T C(q(t), \dot{q}(t), u(t))\, dt,$$

5. Simulation Results

Simulation results shows that DMOC is applied successfully to the compass biped as below.

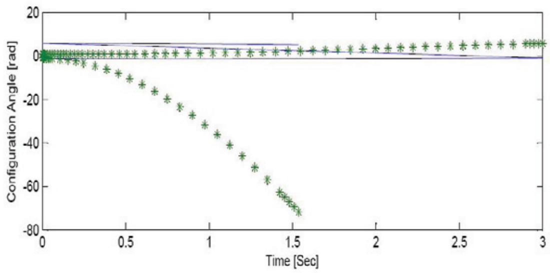

Fig. 2. Trajectories for the compass biped using DMOC

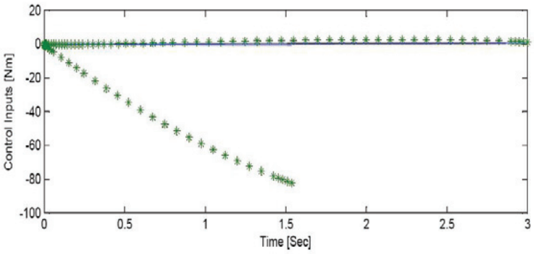

Fig. 3. Control inputs for the compass biped using DMOC

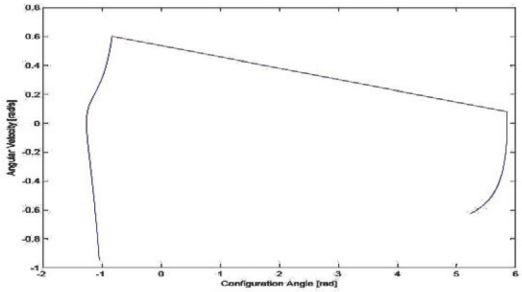

Fig. 4. Phase portraits for compass biped for single step using DMOC

6. Conclusion & Future Work

DMOC has been validated as a useful tool for SHMS's, especially those of low dimension, by formulating optimal control generation as a constrained nonlinear optimization problem. Using the compass gait biped, we have demonstrated that solving DMOC optimization problems to assess optimality in the performance of an existing control policy, as well as solving Simple Hybrid DMOC optimization problems in order to design locally optimal hybrid orbits, provides valuable insights into possible control strategies. Adding complexity to the system constraints (making them nonholonomic for instance) and contact conditions (examining compliant patch contacts for instance) provides challenging important problems.

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