University of Jeddah
Faculty of Engineering
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Control Systems (ECE 331)

Modeling in Time Domain – A State Space Modeling - II

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Applying State Space Representation :

The first step in representing a system is to select the state vector, which must be chosen according to the following considerations:

[1] A minimum number of state variables must be selected as components of the state vector. This minimum number of state variables is sufficient to describe completely the state of the system.

For Example: How to know the minimum no. of state variables to select? Typically, the minimum no. of required equals the order of the differential equation describing the system. If a third-order differential equation describes the system, then three simultaneous, first order differential equations are required along with three state variables.

From the perspective of the transfer function, the order of the differential equation is the order of the denominator of transfer function after canceling common factors in the numerator and denominator.

In most cases, the no. of state variables is to count the no. of independent energy-storage elements in the system.

Applying State Space Representation : [1] Conti…

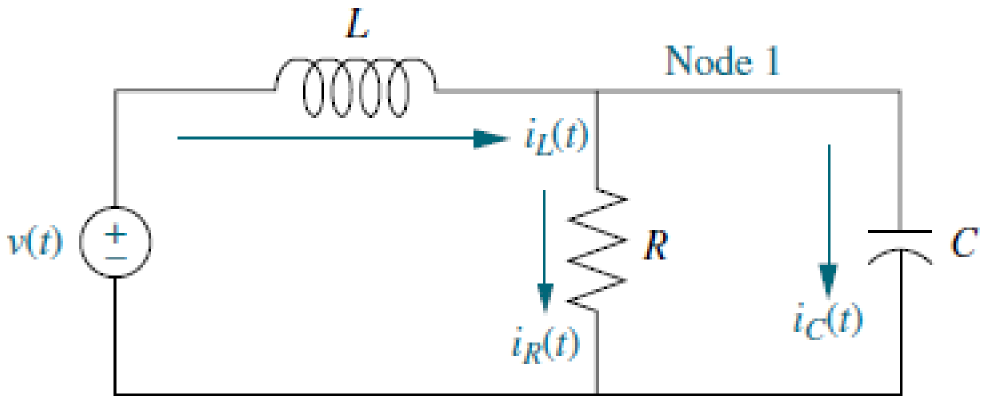
Special Case: If too few state variables are selected, it may be impossible to write particular output equations, since some system variables cannot be written as a linear combination of the reduced no. of state variables. In many cases, it may be impossible even to complete the writing of the state equations, since the derivatives of the state variables cannot be expressed as linear combinations of the reduced no. of state variables.

If you selected the minimum no. of state variables but they are not linearly independent, at best you may not be able to solve for all other system variables. At worst you may not be able to complete the writing of the state equations.

[2] The components of the state vector (that is, this minimum number of state variables) must be linearly independent.

For Example: The voltage across inductor, v_L is linearly independent of the current through the inductor, i_L since, $v_L = {}^{L\,di_L}/_{dt}$. Thus, v_L cannot be evaluated as a linear combination of the current, i_L.

Example :



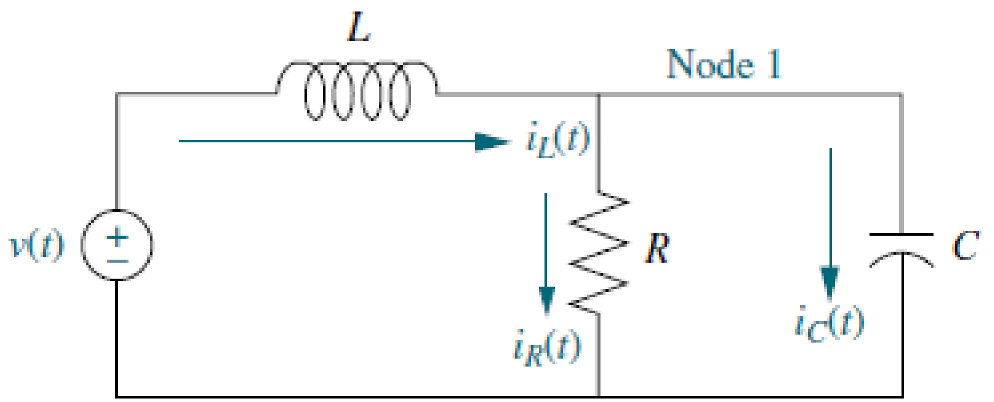
Step 1 Label all of the branch currents in the network. These include i_L, i_R, and i_C, as shown in Figure 3.5.

Step 2 Select the state variables by writing the derivative equation for all energy-storage elements, that is, the inductor and the capacitor. Thus,

$$C\frac{dv_C}{dt} = i_C \tag{3.22}$$

$$L\frac{di_L}{dt} = v_L \tag{3.23}$$

Applying State Space Representation :



Since i_C and v_L are not state variables, our next step is to express i_C and v_L as linear combinations of the state variables, v_C and i_L, and the input, $v(t)$.

Step 3 Apply network theory, such as Kirchhoff's voltage and current laws, to obtain i_C and v_L in terms of the state variables, v_C and i_L. At Node 1,

$$\begin{aligned} i_C &= -i_R + i_L \\ &= -\frac{1}{R}v_C + i_L \end{aligned} \tag{3.24}$$

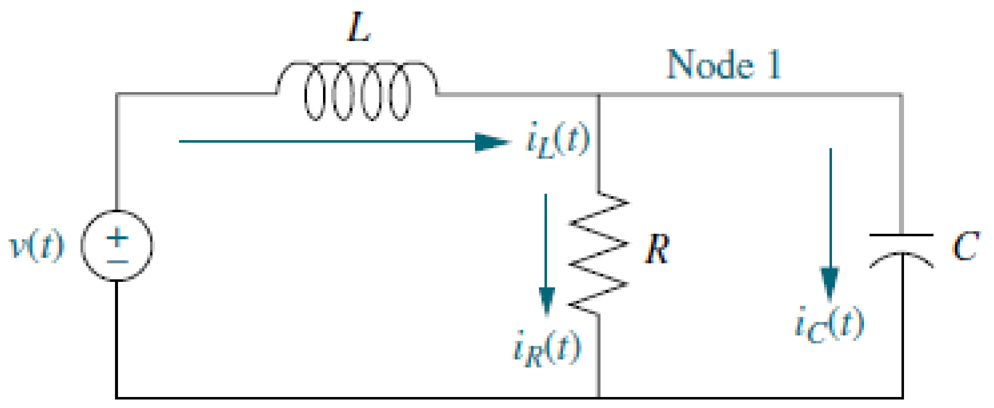
which yields i_C in terms of the state variables, v_C and i_L.

Around the outer loop,

$$v_L = -v_C + v(t) \tag{3.25}$$

which yields v_L in terms of the state variable, v_C, and the source, $v(t)$.

Applying State Space Representation :



Step 4 Substitute the results of Eqs. (3.24) and (3.25) into Eqs. (3.22) and (3.23) to obtain the following state equations:

$$C\frac{dv_C}{dt} = -\frac{1}{R}v_C + i_L \quad (3.26a)$$

$$L\frac{di_L}{dt} = -v_C + v(t) \quad (3.26b)$$

or

$$\frac{dv_C}{dt} = -\frac{1}{RC}v_C + \frac{1}{C}i_L \quad (3.27a)$$

$$\frac{di_L}{dt} = -\frac{1}{L}v_C + \frac{1}{L}v(t) \quad (3.27b)$$

Step 5 Find the output equation. Since the output is $i_R(t)$,

$$i_R = \frac{1}{R}v_C \quad (3.28)$$

The final result for the state-space representation is found by representing Eqs. (3.27) and (3.28) in vector-matrix form as follows:

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -1/(RC) & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} v(t) \quad (3.29a)$$

$$i_R = [1/R \quad 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix} \quad (3.29b)$$

where the dot indicates differentiation with respect to time.

Thank You !

Reference: Control System Engineering by Norman S. Nise