جـــامعـــة جـــــدة
University of Jeddah

Faculty of Engineering
Department of Electrical & Computer Engineering

Control Systems (ECE 331)

Modeling in Time Domain – A State Space Modeling - I

Ankit Patel
majorankit@gmail.com
http://majorankit.wix.com/majorankit

Difference : Frequency (Classical) & Time (State Space) Domain Analysis :

Frequency or Classical Approach:

The Frequency domain technique is based on converting a system's equation into differential equation to a transfer function, thus generating a mathematical model of the system that algebraically relates a representation of the output to a representation of a input.

The primary disadvantage of classical approach is its limited applicability: It can be applied to linear, time-invariant systems or systems that can be approximately as such.

A major advantage of frequency domain technique is that they rapidly provide stability & transient response information. Thus, we can immediately see the effects of varying system parameters until an acceptable design is met.

Difference : Frequency (Classical) & Time (State Space) Domain Analysis :

Time Domain or State Space Approach:

The state space or modern or time domain approach is a unified method for modeling, analyzing and designing a wide range of systems. For example: This approach can be used to represent nonlinear systems have backlash, saturation, and dead zone. Also, it can handle easily systems with nonzero initial conditions. The time domain approach can be used to represent systems with a digital computer in the loop or to model systems for digital simulation. With simulated systems, system response can be obtained for changes in system parameters – an important design tool. This approach is also attractive because of the availability of numerous state-space approach packages for the personal computer.

The time-domain approach can also be used for the same class of systems modeled by the classical approach. This alternate model gives the control systems designer another perspective from which to create a design. While the state-space approach can be applied to a wide range of systems, it is not as intuitive as the classical approach.

The General State Space Representation :

Linear Combinations: A linear combination of n variables, x_i, for $i = 1$ to n, is given by the following sum, $S = K_n x_n + K_{n-1} x_{n-1} + \ldots + K_1 x_1$; Where K_i is a constant.

Linear Independent: A set of variables is said to be linearly independent if none of the variables can be written as a linear combination if the others. For example: given x_1, x_2 and x_3, if $x_2 = 5x_1 + 6x_3$, then the variables are not linearly independent, since one of them can be written as a linear combination of the other two. Now, what must be true so that one variable cannot be written as a linear combination of the other variables?. Consider the example-> $K_2 x_2 = K_1 x_1 + K_3 x_3$. If no $x_i = 0$, then any x_i can be written as a linear combination of other variables, unless all $K_i = 0$. Formally, then, variables x_i, for i = 1 to n, are said to be linearly independent if their linear combination, S, equals zero only if every $K_i = 0$ and no $x_i = 0$.

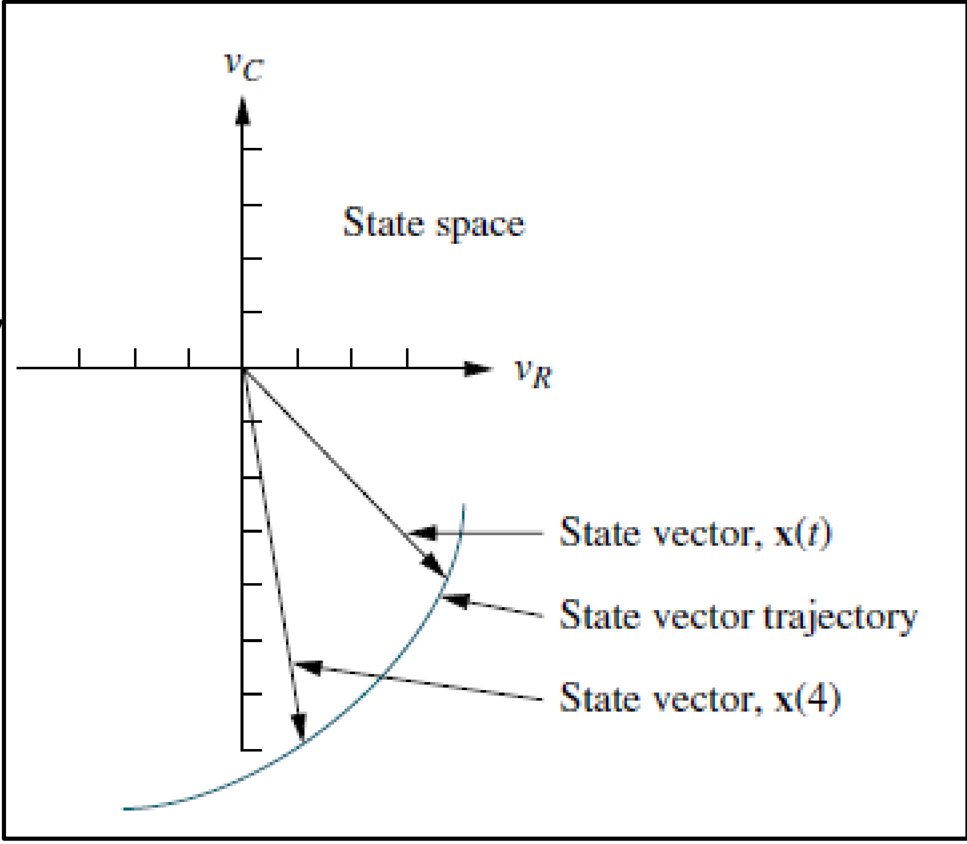
System Variable: Any variable that responds to an input or initial conditions in a systems.

State Variables: The smallest set of linearly independent system variables such that the values of the members of the set at time t_0 along with known forcing functions completely determine the value of all system variables for all $t \geq t_0$.

State Vector: A vector whose elements are the state variables.

The General State Space Representation : Conti…

State Space: The n-dimensional space whose axes are the state variables. Fig. shows the state variables are assumed to be a resistor voltage, v_R, and a capacitor voltage, v_C. These variables form the axes of the state space. A trajectory can be thought of as being mapped out by the state vector, X(t), for a range of t. Also shown is the state vector at the particular time t=4.



State Equations: A set of n simultaneous, first order differential equations with n variables, where the n variables to be solved are the state variables.

Output Equation: The algebraic equation that expresses the output variables of a system as linear combinations of the state variables and the inputs.

The General State Space Representation : Conti…

A system is represented in the state space equations as under:

$$\dot{X} = Ax + Bu$$ ----------State Equation

$$y = Cx + Du$$ -------- Output Equation

X = State vector

$\dot{X}$ = Derivation of the state vector with respect to time

y = Output vector

u = Input or Control vector

A = System matrix

B = Input matrix

C = Output matrix

D = Feedforward matrix

Some Observations regarding State Space Approach :

[1] We select a particular subset of all possible system variables and call the variables in this subset state variables.

[2] For an nth order system, we write *n* simultaneous, first order differential equations in terms of the state variables. We call this system of simultaneous differential equations state equations.

[3] If we know the initial conditions of all of the state variables at t_0 as well as the system input for $t \geq t_0$, we can solve the simultaneous differential equations for the state variables for $t \geq t_0$.

[4] We algebraically combine the state variables with the system's input and find all the other system variables for $t \geq t_0$. We call this algebraic equation the output equation.

[5] We consider the state equations and the output equations a viable representation of the system. We call this representation of the system a state-space representation.

Thank You !