University of Jeddah
Faculty of Engineering
Department of Electrical & Computer Engineering

Control Systems (ECE 331)

Transient Response Analysis

Ankit Patel
majorankit@gmail.com
http://majorankit.wix.com/majorankit

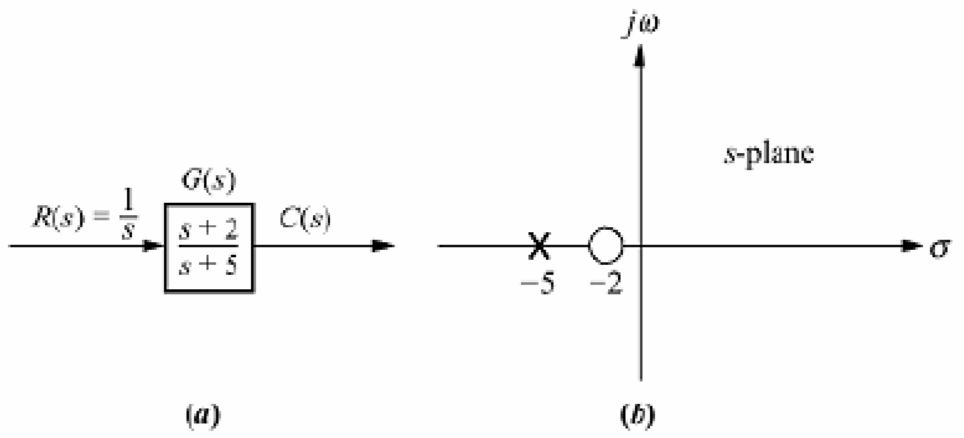
Importance of Poles and Zeros in Control Theory :

The poles of a transfer function are (a) the values of the Laplace transform variable, *s,* that cause the transfer function to become infinite or (b) any roots of the denominator of the transfer function that are common roots of the numerator.

The zeros of a transfer function are (a) the values of the Laplace transform variable, *s*, that cause the transfer function becomes zero, or (b) any roots of the numerator of the transfer function that are common roots of the denominator.

$$C(s)=\frac{s+2}{s(s+5)}=\frac{A}{s}+\frac{B}{s+5}=\frac{2/5}{s}+\frac{3/5}{s+5}$$ where $A=\left.\frac{s+2}{s}\right|_{s\to 0}=\frac{2}{5}$ and $B=\left.\frac{s+2}{s}\right|_{s\to -5}=\frac{3}{5}$

Thus, $$c(t)=\frac{2}{5}+\frac{3}{5}e^{-5t}$$



Importance of Poles and Zeros in Control Theory :

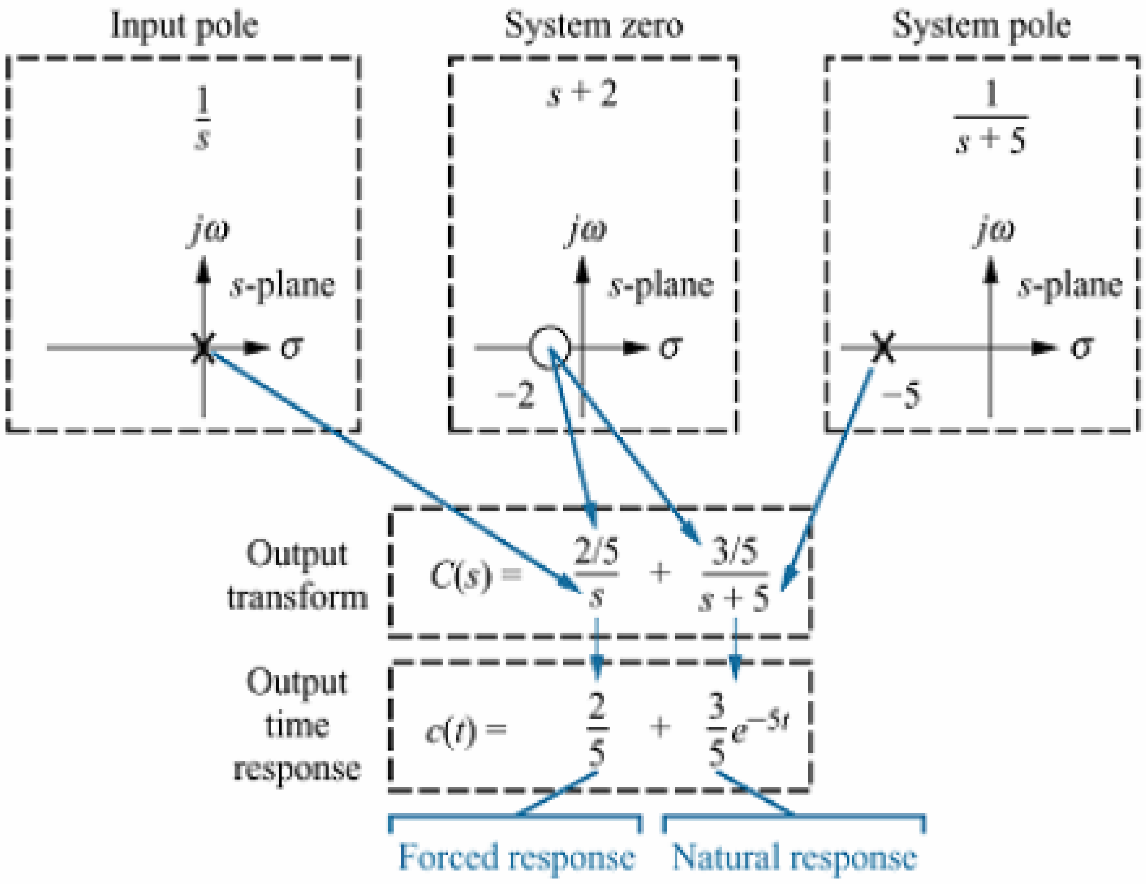
Conclusion:

[1] A pole of the input function generates the form of the forced response (that is, the pole at the origin generated a step function at the output).

[2] A pole of the transfer function generates the form of the natural response (that is, the pole at -5 generated e^{-5t}).

[3] A pole on the real axis generates an exponential response of the form $e^{-\alpha t}$. Where α is the pole location on the real axis. Thus, the farther to the left a pole is on the negative real axis, the faster the exponential transient response will decay to zero (again, the pole at -5 generated e^{-5t}).

[4] The zeros and poles generate the amplitudes for both the forced and natural response.

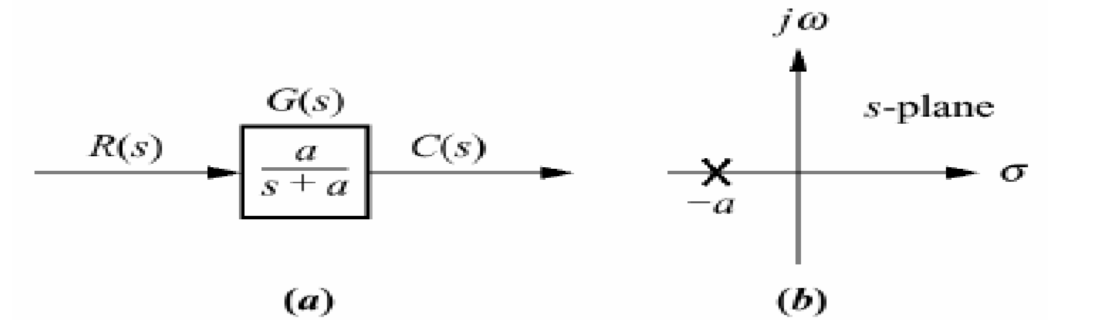


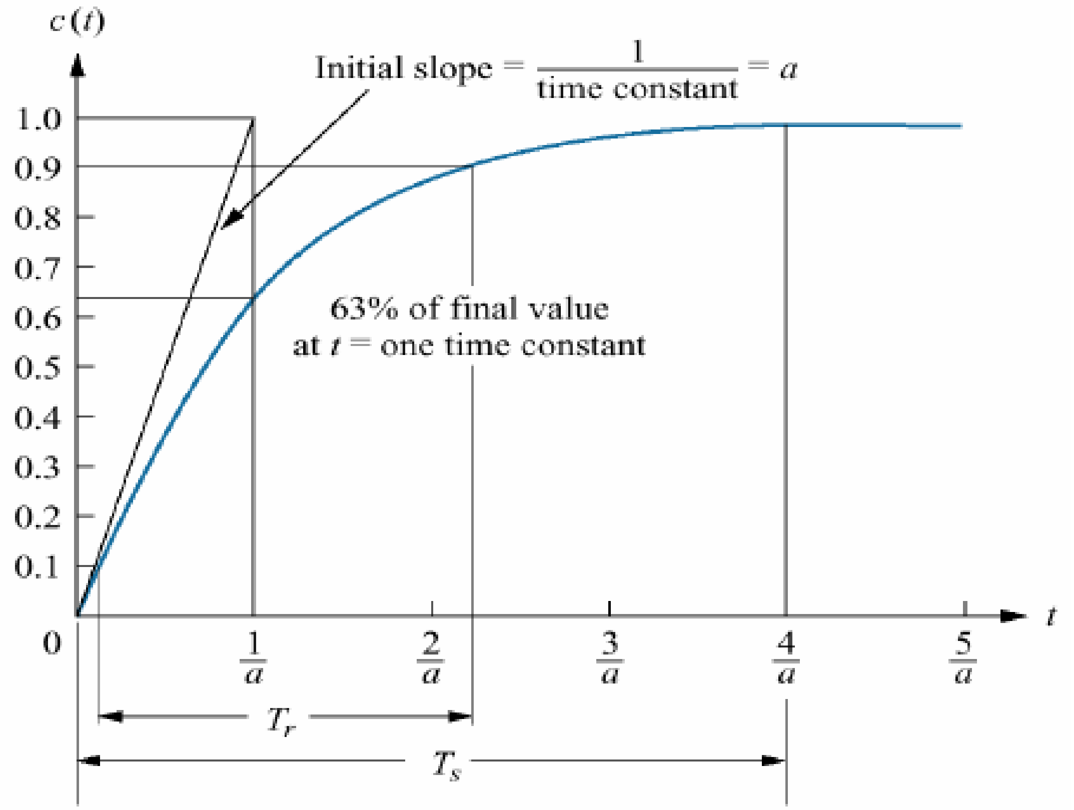
Transient Response for First Order Systems :

A first order without zeros can be described by the transfer function given in fig. (a). If the input is a unit step, where $R(s) = 1/s$, the Laplace transform of the step response is $C(s)$, where $C(s) = R(s)G(s) = \frac{a}{s(s+a)}$. Taking the inverse Laplace transform, the step response is given by

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

Where the input pole at the origin generated the forced response $c_f(t) = 1$, and the system pole at –a, shown in (b).





Transient Response Specifications:

[1] Time Constant: We call $1/a$, the time constant of the response. From the above equations, the time for e^{-at} to decay to 37% of its initial time. Alternately, the time constant is the time it takes for the step response to rise to 63% of its final value. Thus, we call the parameter *a* as exponential frequency. $c(t)\big|_{t=1/a} = 1 - e^{-at}\big|_{t=1/a} = 1 - 0.37 = 0.63$

$$e^{-at}\big|_{t=1/a} = e^{-1} = 0.37$$

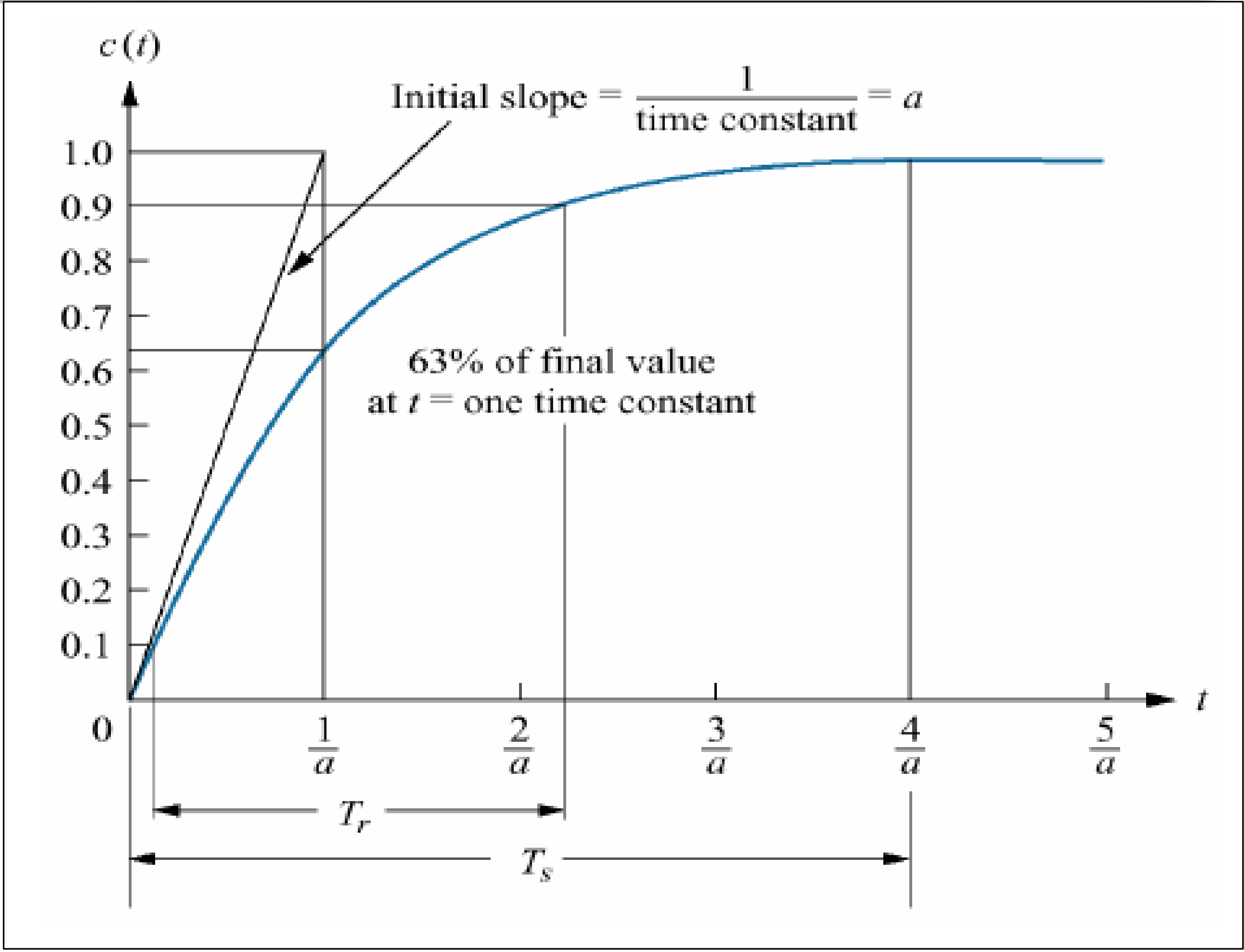
Transient Response Specifications:

[2] Rise Time T_r: Rise time is defined as the time for the waveform to go from 0.1 to 0.9 of its final value. For the given example, c(t)=0.9 and c(t)=0.1, so

$$T_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$$

[3] Settling Time T_s: Settling time is defined as the time for the response to reach and stay within 2% of its final value. Letting c(t)=0.98, the settling time to be

$$T_s = \frac{4}{a}$$



Transient Response for Second Order Systems :

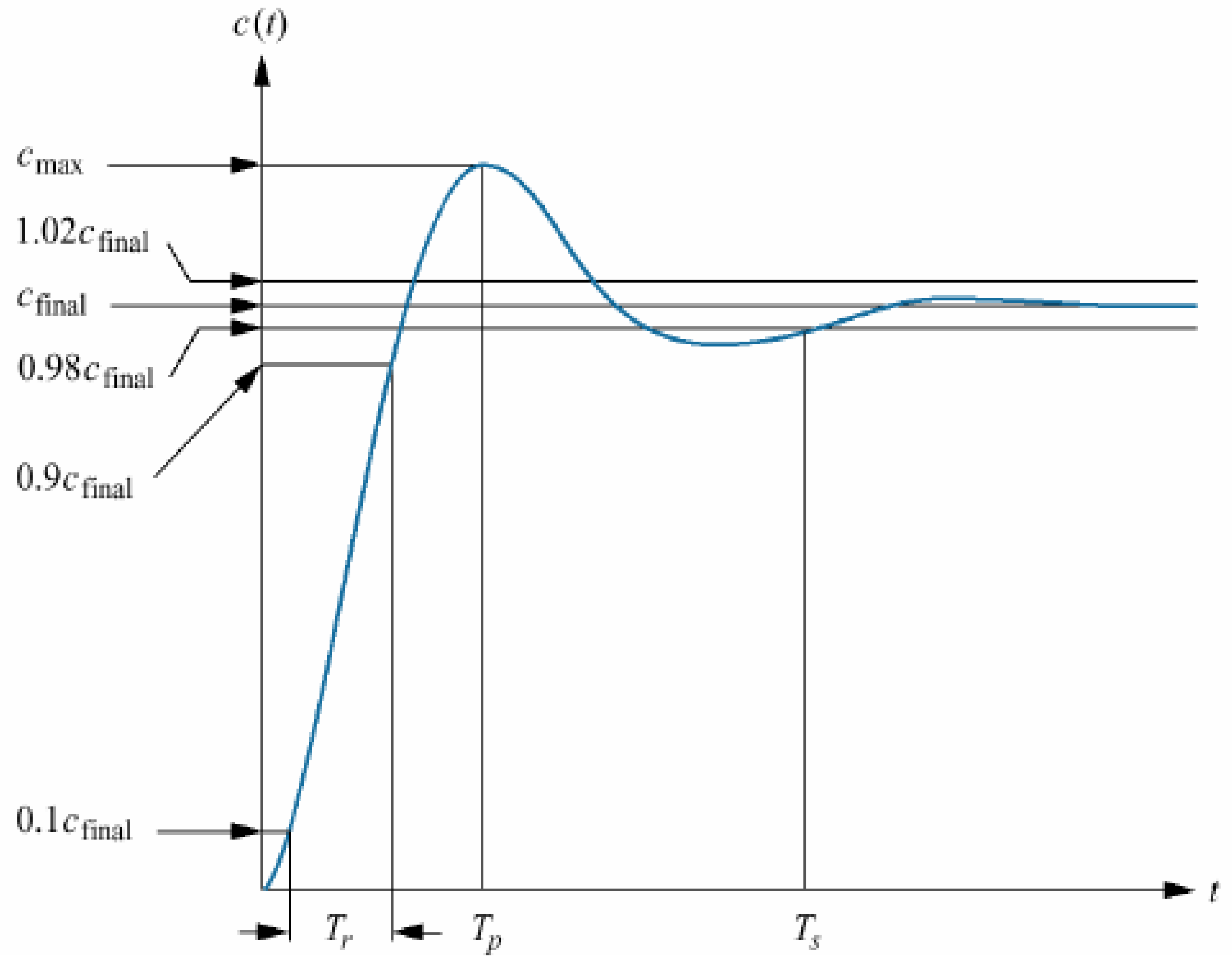
Transient Response Specifications:

[1] Natural Frequency ω_n: The natural frequency of the second order system is the frequency of oscillation of the system without damping. For example, the frequency of oscillation of a series RLC circuit with the resistance shorted would be natural frequency.

[2] Damping Ratio ξ: We define the damping ratio, ξ as,

$$\xi = \frac{Exponential\ decay\ frequency}{Natural\ frequency\ (rad\ per\ second)}$$

$$= \frac{1}{2\pi}\frac{Natural\ period\ (second)}{Exponential\ time\ constant}$$



Let us now consider the general system, $G(s) = \frac{b}{s^2+as+b}$

Compare with general second order system, $G(s) = \frac{\omega^2{}_n}{s^2+2\xi\omega_n s+\omega^2{}_n}$, we get $b = \omega^2{}_n$ and

$a = 2\xi\omega_n$

[3] Rise Time T_r: Rise time is defined as the time for the waveform to go from 0.1 to 0.9 of its final value.

[4] Peak Time T_p: The time required to reach the first, or maximum peak.

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

[5] Percentage Overshoot %OS: The amount that the waveform overshoots the steady-state, or final, value at the peak time, expressed as a percentage of the steady state value.

$$\%OS = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$$

[6] Settling Time T_s: The time required for the transient's damped oscillation to reach and stay within ±2% of the steady state value.

$$T_s = \frac{4}{\zeta\omega_n}$$

Thank You !