University of Jeddah

Faculty of Engineering
Department of Electrical & Computer Engineering (ECE)

Control Systems (ECE 331)

Experiment No: 05

"Simulation of a Prototype Second Order System"

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Control Systems ECE 331

Experiment 05

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1 Introduction

1.1 Time Response:

It is an equation or a plot that describes the behavior of a system and contains many information about it with respect to time response specification as overshooting, settling time, peak time, rise time and steady state error. Time response is formed by the transient response and the steady state response.

$$\text{Time Response} = \text{Transient Response} + \text{Steady State Response}$$

Transient time response (Natural response) describes the behavior of the system in its first short time until it arrives the steady state value and this response will be our study focus.If the input is step function the output or the response is called step time response and if the input is ramp, the response is called ramp time response etc.

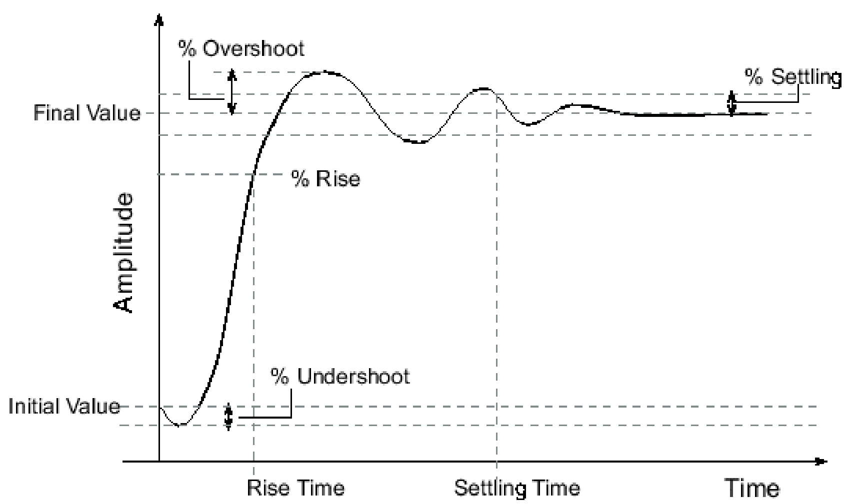

Figure 1: Basic Time Response Information

1.1.1 Step Time Response Specifications:

1. **Percentage Overshoot (%OS):** is the maximum fraction by which the response overshoots the steady state value expressed as a percentage. This characteristic is not found in a first order system and found in higher one for underdamped step response.

2. **Settling Time (Ts):** is the time required to fall within a certain percentage of the steady state value for a step input. For example the amount of time required for the step response to reach and stay within 2% of the steady state value OR in other words we can define it as the smallest amount of time required to reach the steady state value.

3. **Peak Time (Tp):** is the time required for the underdamped step response to reach the first maximum peak.

4. **Rise Time (Tr):** is the time required for the step response to go from 10% to 90% of the final value.

5. **Steady State Error (Ess):** is the difference between the input and the output of a system after the natural response has finished.

6. **DC Gain:** The DC gain is the ratio of the steady state step response to the magnitude of a step input. For example if your input is step function with amplitude = 1 and found the step response output = 5 then the DC gain = 5/1 = 5. In other words it is the value of the transfer function when s=0.

1.1.2 Types of Step Time Response:

We know that the system can be represented by a transfer function which has poles (values make the denominator equal to zero), depending on these poles the step response divided into four cases:

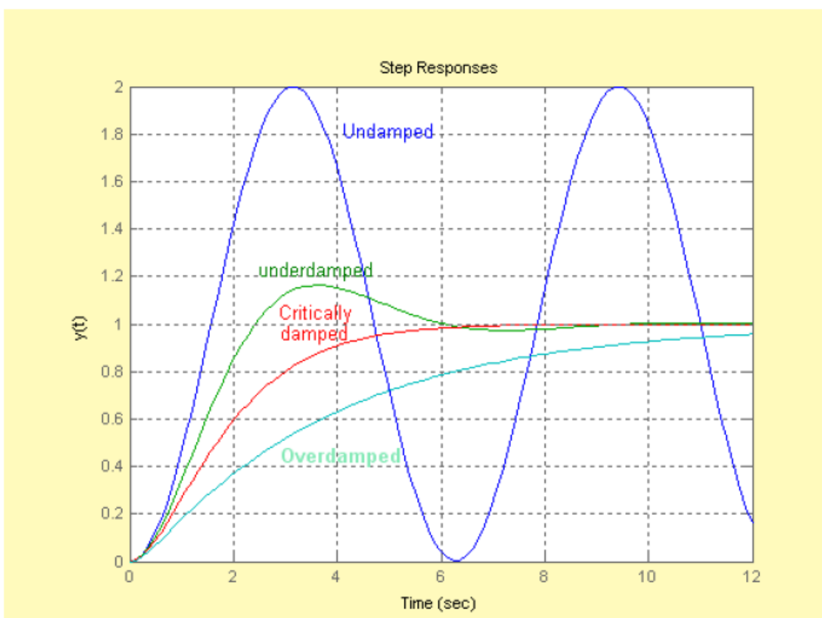

Figure 2: Response Analysis of Step Input

1. **Underdamped Response:** In this case the response has an overshooting with a small oscillation which results from **complex poles** in the transfer function of the system.

2. **Critically Response:**In this case the response has no overshooting and reach the steady state value (final value) in the fastest time. In other words it is the fastest response without overshooting and is resulted from the existence of **real & repeated poles** in the transfer function of the system.

3. **Overdamped Response:**In this case no overshooting will appear and reach the final value in a time larger than critically case. This response is resulted from the existence of **real & distinct poles** in the transfer function of the system.

4. **Undamped Response:**In this case a large oscillation will appear at the output and will not reach a final value and this because of the existence of **imaginary poles** in the transfer function of the system and the system in this case is called **"Marginally stable"**.

2 Characteristic of Second Order System:

The general form of second order system is:

$$G(s) = \frac{a}{(s^2+bs+c)} \quad G(s) = \frac{K_{dc}\omega_n^2}{(s^2+2\xi\omega_n s+\omega_n^2)}$$

Natural Frequency (ω_n)**:** is the frequency of oscillation of the system without damping.

Damping Ratio ξ**:** $\xi = \frac{b}{2\omega_n}$

Note that the system has a pair of complex conjugate poles at:

$S = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -\sigma \pm j\omega$; where ω is the damped frequency of oscillation

DC Gain: $= K_{dc} = \frac{a}{c}$

Percentage Overshoot $\%OS$**:** $= e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100$

Settling Time T_s**:** $T_s = \frac{4}{\xi\omega_n} = \frac{4}{\sigma}$

Peak Time T_p**:** $T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

Now taking an example of second order system, we will be able to find out its response graph. The procedure for the same as under:

A given transfer function is:

$$H(s) = \frac{9}{s^2+2s+9}$$

Its MATLAB code is:

```
num=[9];
den=[1 2 9];
step(num,den)
```

Also we can write,

```
sys=tf(num,den);
step(Sys)
```

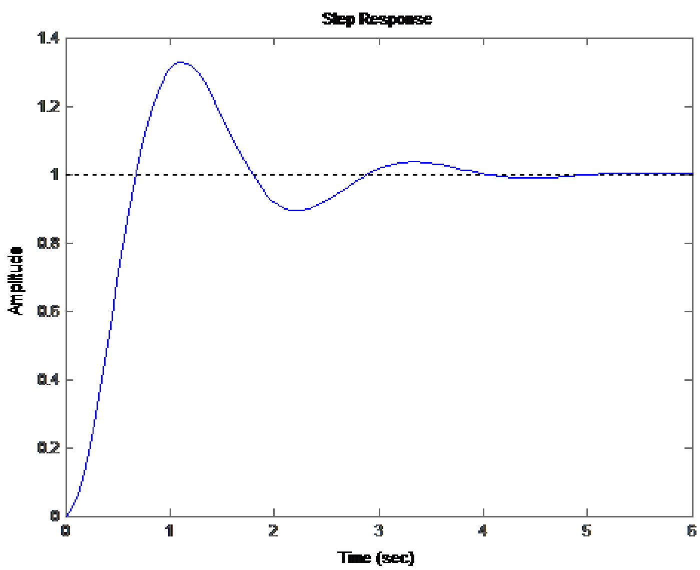

Figure 3: Response Analysis of Step Input for given Transfer Function

3 Second Order Prototype System

Although true second-order control systems are rare in practice, their analysis generally helps to form a basis for the understanding of analysis and design of higher-order systems. The second-order system is defined as the prototype second-order system.

The closed loop transfer function of the system is:

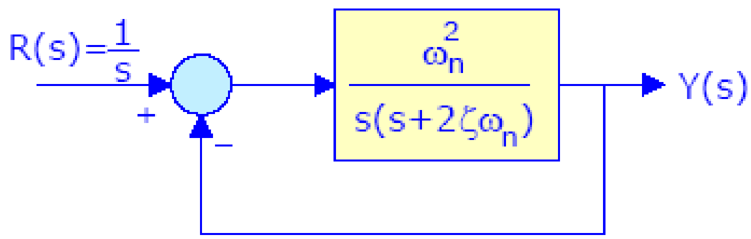

Figure 4: Prototype of Second Order System

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$$

For a unit step input, $R(s) = \frac{1}{S}$, the output response of the system is

$$Y(S) = \frac{\omega_n^2}{S(S^2+2\xi\omega_n S+\omega_n^2)}$$

The dynamic behavior of second-order systems can be described in terms of the two parameters ξ and ω
where, ω_n = Undamped Natural Frequency
ξ = Damping Ratio

As per the response analysis, there are four cases for the given transfer function as under:

`CASE::01 Underdamped case`

It means $0 < \xi < 1$, In this case, the system output for a unit-step input, can be written as:

$$Y(S) = \frac{\omega_n^2}{S(S+\xi\omega_n \pm j\omega_n\sqrt{(1-\xi^2)})}$$

$$y(t) = 1 - \frac{1}{\sqrt{(1-\xi^2)}}e^{-\xi\omega_n t}sin(\omega_n\sqrt{(1-\xi^2)}t+\theta); \text{ where } \theta = cos^{-1}\xi$$

Note that the system has a pair of complex conjugate poles at:

$$S = -\xi\omega_n \pm j\omega_n\sqrt{(1-\xi^2)} = -\sigma \pm j\omega \text{ as shown in figure.}$$

The maximum overshoot and the time it occurs (peak time) are given by:

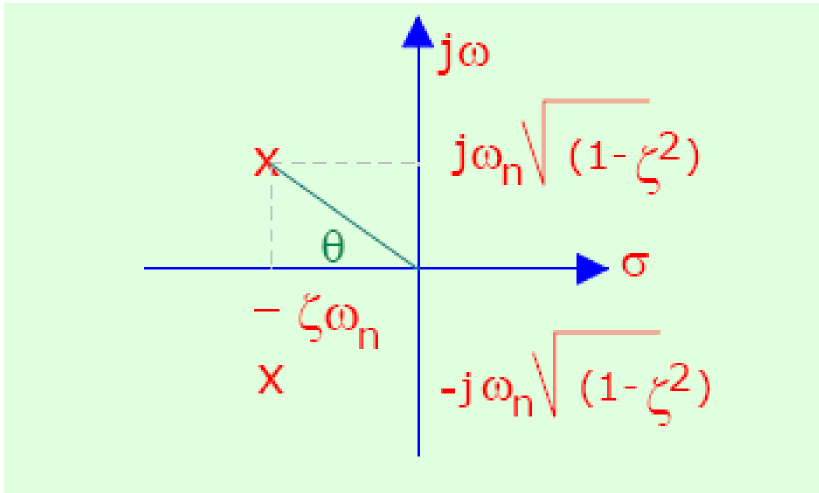

Figure 5: Prototype of Second Order System for Underdamped System

$$OS_{max} = e^{\frac{-\pi\xi}{\sqrt{(1-xi^2)}}} \text{ and } T_p = \frac{\pi}{\omega_n\sqrt{(1-xi^2)}}$$

The underdamped system response is a damped sinusoid with an exponential envelope whose time constant is equal to the reciprocal of the pole's real part. The radian frequency of the sinusoid is equal to the imaginary part of the pole.

`CASE::02 Critically Damped System`

It means $\xi = 1$, In this case, the system output for a unit-step input, can be written as:

$$Y(S) = \frac{\omega_n^2}{S(S^2+2\xi\omega_n S+\omega_n^2)} = \frac{\omega_n^2}{S(S+\omega_n)^2}$$
$$y(t) = 1 - e^{-\omega_n t}(1+\omega_n t)$$

Note that the system has 2 repeated real poles at $S = -\omega_n$ as shown in figure.

`CASE::03  Overdamped System`

It means $\xi > 1$, the system output for the unit step function can be written as:

$$Y(S) = \frac{\omega_n^2}{S(S^2+2\xi\omega_n S+\omega_n^2)} = \frac{\omega_n^2}{S(S+\xi\omega_n \pm \omega_n\sqrt{(\xi^2-1)})}$$
$$y(t) = 1 + \frac{\omega}{2\sqrt{(\xi^2-1)}}\left(\frac{e^{-S_1 t}}{S_1} - \frac{e^{-S_2 t}}{S_2}\right)$$
$$S_1 = \xi\omega_n + \omega_n\sqrt{(}\xi^2 - 1) \text{ and } S_2 = \xi\omega_n - \omega_n\sqrt{(}\xi^2 - 1)$$

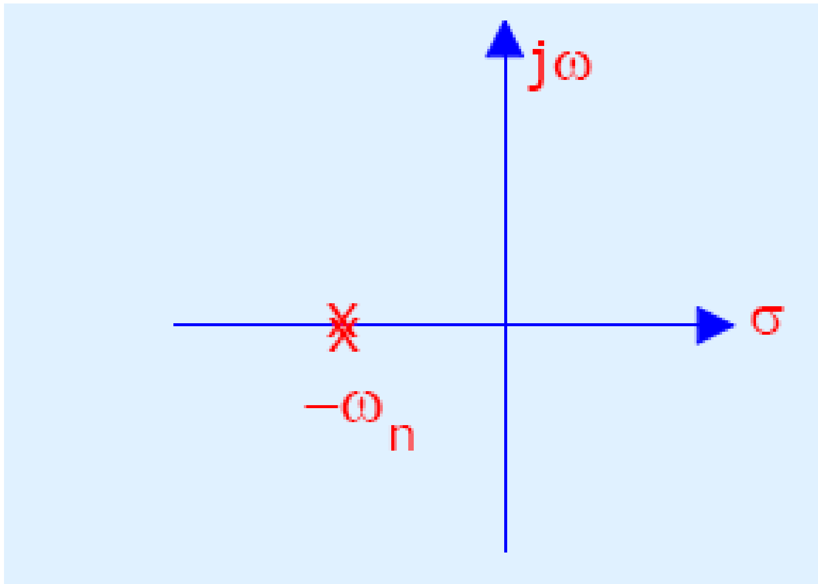

Figure 6: Prototype of Second Order System for Critically System

Note that the system has 2 distinct real poles at:
$S = \xi\omega_n \pm \omega_n\sqrt{(\xi^2 - 1)}$ as shown in figure.

`CASE::04  Undamped System`

It means, $\xi = 0$, the system output for a unit-step input, can be written as:

$$Y(S) = \frac{\omega_n^2}{S(S^2+\omega_n^2)} = \frac{\omega_n^2}{S(S^2 \pm j\omega_n)}$$
$$y(t) = 1 - cos(\omega_n t)$$

Note that the system has a pair of imaginary poles as shown in figure.

The system response is an undamped sinusoid with radian frequency equal to the imaginary part of the poles.

Typical step responses of the prototype second-order system, for the four cases considered, are shown in figure 9. Notice that the critically damped case is the division between the overdamped cases and the underdamped cases and is the fastest response without overshoot.

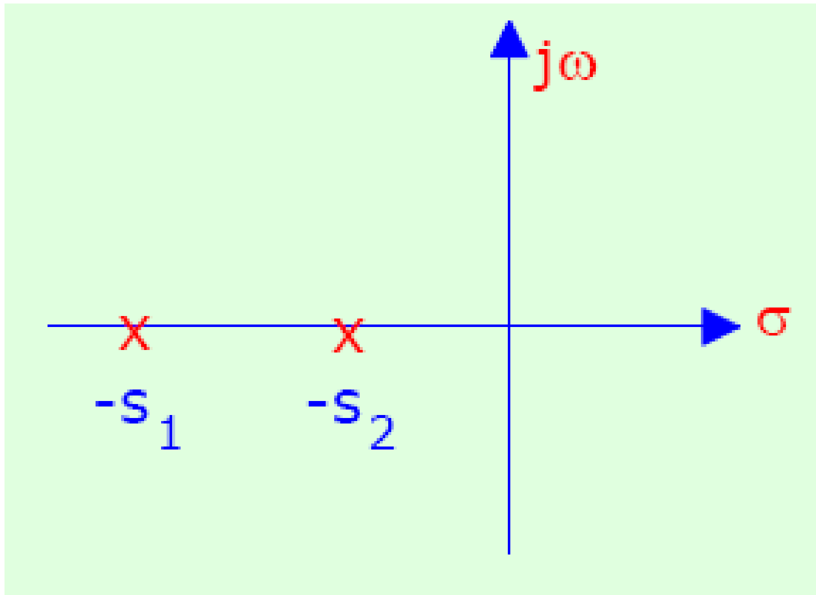

Figure 7: Prototype of Second Order System for Overdamped System

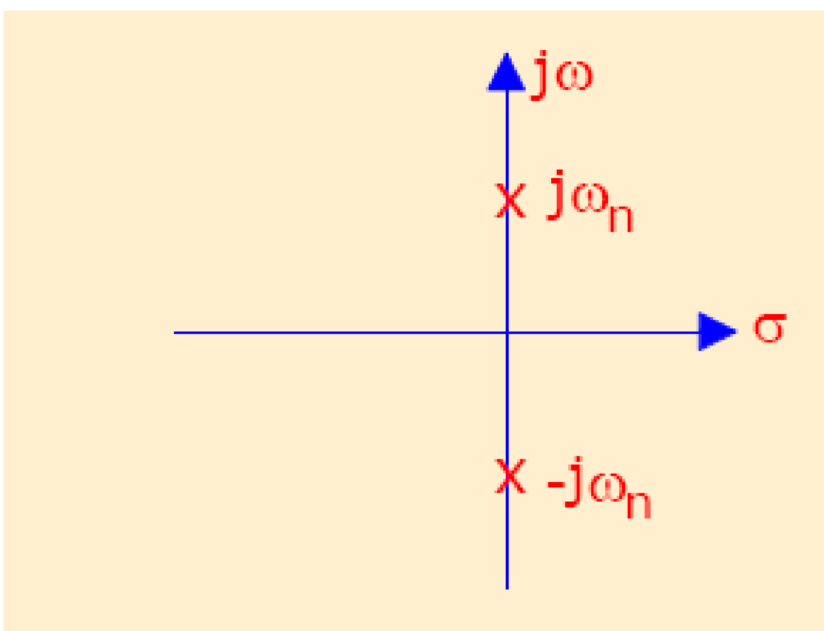

Figure 8: Prototype of Second Order System for Undamped System

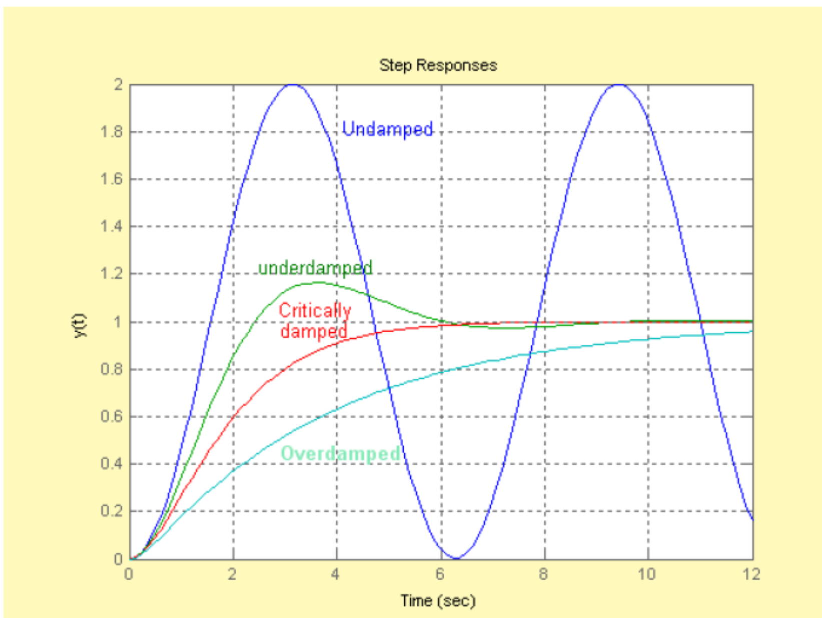

Figure 9: Step responses of a prototype second-order system