University of Jeddah
Faculty of Engineering
Department of Electrical & Computer Engineering

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Electromagnetic Fields (ECE 308)

Lecture 6 – Electrostatics - III

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Coulomb's Law:

Electric Field Due to Multiple Point Charges:

The Coulomb's Law for the field E due to a single point charge can be extended to multiple charges. As shown in fig. we taken two point charges q1 and q2 with position vectors R1 and R2. The electric field E is to be evaluated at a point P with position vector R. At P, the Electrical field E1 due to q1 and the distance between q1 and P, replaced with |R-R1| and the unit vector R replaced with (R-R1)/|R-R1|, will be given by:

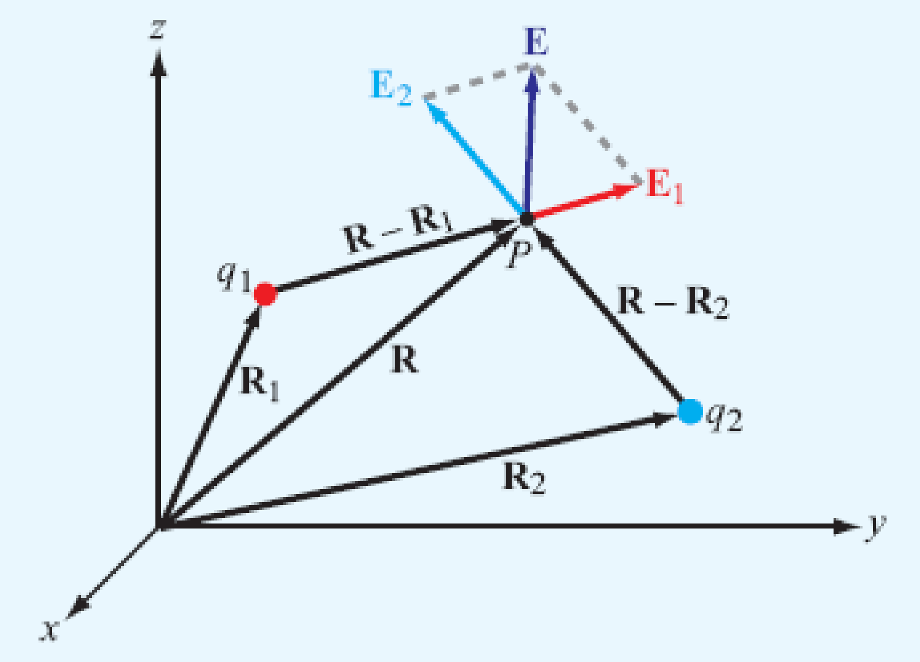

Figure 4-4 The electric field **E** at *P* due to two charges is equal to the vector sum of $\mathbf{E}_1$ and $\mathbf{E}_2$.

$$\mathbf{E}_1 = \frac{q_1(\mathbf{R}-\mathbf{R}_1)}{4\pi\epsilon|\mathbf{R}-\mathbf{R}_1|^3}$$

Coulomb's Law:

Electric Field Due to Multiple Point Charges:

Similarly, the electric field at P due to q2 alone is given by:

$$\mathbf{E}_2 = \frac{q_2(\mathbf{R} - \mathbf{R}_2)}{4\pi\epsilon|\mathbf{R} - \mathbf{R}_2|^3}$$

The electric field obeys the principle of linear superposition, hence

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$
$$= \frac{1}{4\pi\epsilon}\left[\frac{q_1(\mathbf{R} - \mathbf{R}_1)}{|\mathbf{R} - \mathbf{R}_1|^3} + \frac{q_2(\mathbf{R} - \mathbf{R}_2)}{|\mathbf{R} - \mathbf{R}_2|^3}\right]$$

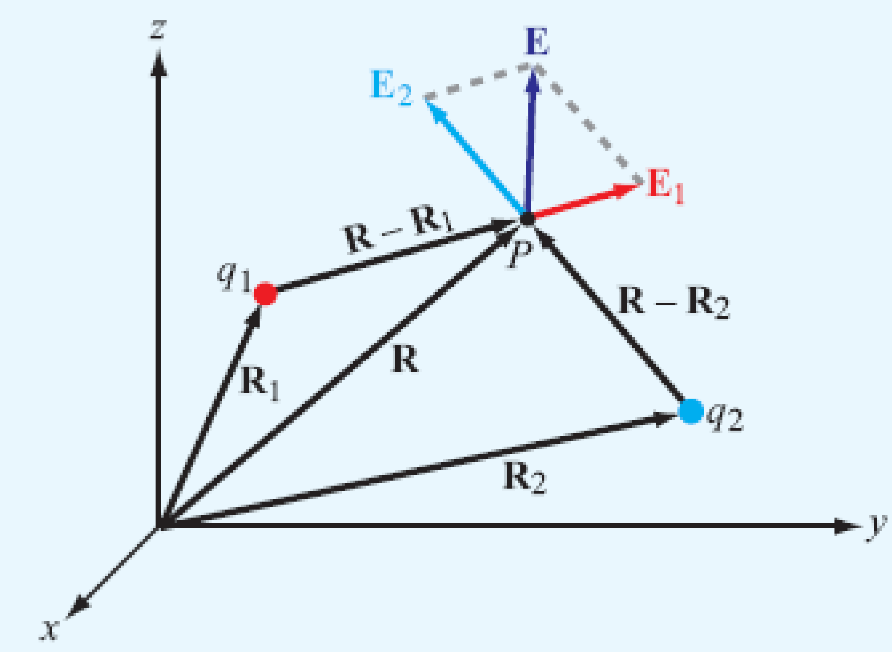

Figure 4-4 The electric field $\mathbf{E}$ at P due to two charges is equal to the vector sum of $\mathbf{E}_1$ and $\mathbf{E}_2$.

In general form, it can be rewritten as:

$$\mathbf{E} = \frac{1}{4\pi\epsilon}\sum_{i=1}^{N}\frac{q_i(\mathbf{R} - \mathbf{R}_i)}{|\mathbf{R} - \mathbf{R}_i|^3}$$

Coulomb's Law:

Electric Field Due to Charge Distribution:

We now extend the results obtained for the field due to discrete point charges to continuous charge distribution. Consider a volume *v'* that contains a distribution of electric charge with volume charge density ρ_v, which may vary spatially within *v'* as shown in fig.

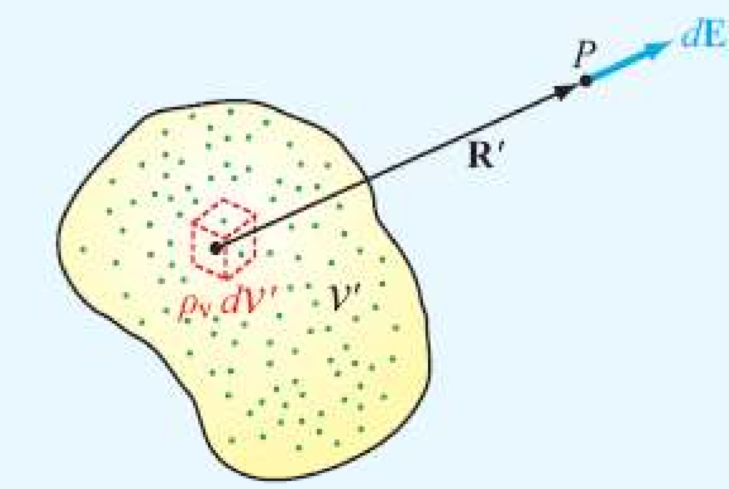

Figure 4-5 Electric field due to a volume charge distribution.

Coulomb's Law:

Electric Field Due to Charge Distribution:

Volume Charge Distribution:

The differential electric field at a point P due to a differential amount of charge dq = ρ_v dv' contained in a differential volume dv' is :

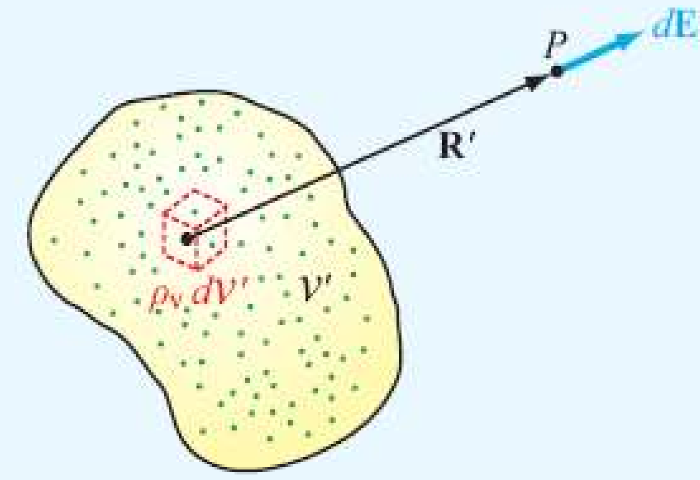

Figure 4-5 Electric field due to a volume charge distribution.

$$d\mathbf{E} = \hat{\mathbf{R}}'\,\frac{dq}{4\pi\epsilon R'^2} = \hat{\mathbf{R}}'\,\frac{\rho_v\, d\mathcal{V}'}{4\pi\epsilon R'^2}$$

Applying the principle of linear superposition , the total electric field **E** is obtained by integrating the fields due to all differential charges in v' is

$$\mathbf{E} = \int_{\mathcal{V}'} d\mathbf{E} = \frac{1}{4\pi\epsilon}\int_{\mathcal{V}'} \hat{\mathbf{R}}'\,\frac{\rho_v\, d\mathcal{V}'}{R'^2}$$

(volume distribution)

Coulomb's Law:

Electric Field Due to Charge Distribution:

Surface & Line Charge Distribution:

If the charge is distributed across a surface S' with surface charge density ρ_s, then dq = ρ_s .dS', and if if it is distributed along a line l' with a line charge density ρ_l, then dq = ρ_s . dl'. Accordingly, the electric fields due to surface and line charge distributions are:

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{S'} \hat{\mathbf{R}}' \frac{\rho_s \, ds'}{R'^2}$$

(surface distribution)

$$\mathbf{E} = \frac{1}{4\pi\epsilon} \int_{l'} \hat{\mathbf{R}}' \frac{\rho_\ell \, dl'}{R'^2}$$

(line distribution)

Gauss's Law:

The Maxwell's equations to confirm the expressions for the electric field implied by Coulomb's Law, and propose alternative techniques for evaluating electric fields induced by electric charge. This can be written By:

$$\nabla \cdot \mathbf{D} = \rho_v ,$$

(differential form of Gauss's law)

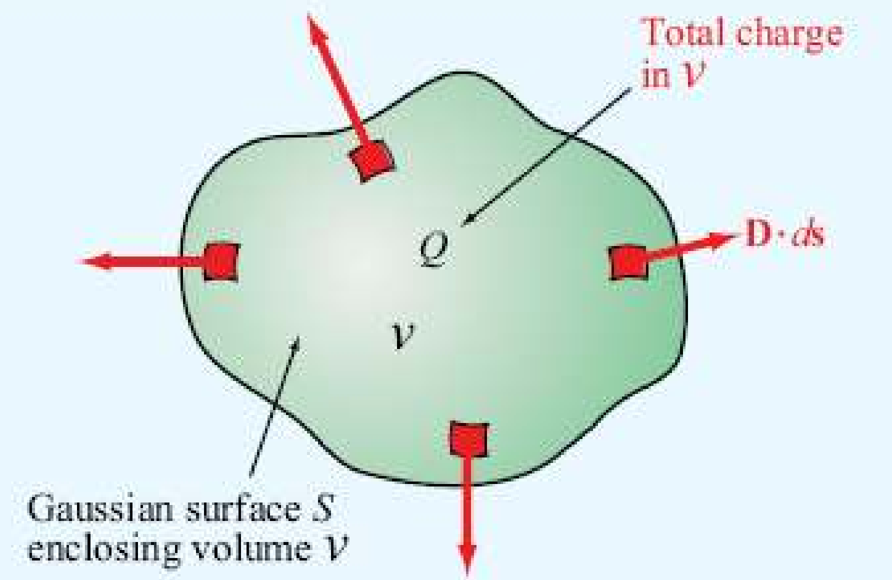

Figure 4-8 The integral form of Gauss's law states that the outward flux of **D** through a surface is proportional to the enclosed charge Q.

Which is referred as differential form of Gauss's Law. The adjective "differential" referred to the fact that the divergence operation involves spatial derivatives. The differential form of Gauss's Law can be converted to an integral form. When solving electromagnetic problems, we often go back and forth between equations in differential and integral form, depends on applications.

Gauss's Law:

The differential form of Gauss's Law is to be convert into integral form by multiply both sides by *dv* and evaluate their integrals over an arbitrary volume *v,* gives

$$\int_{v} \nabla \cdot \mathbf{D}\, d\mathcal{V} = \int_{v} \rho_v \, d\mathcal{V} = Q.$$

Here Q is the total charge enclosed in *v*. **The divergence theorem, states that the volume integral of the divergence of any vector over a volume *v* equals the total outward flux of that vector through the surface S enclosing *v*.** Thus, for the vector **D**,

$$\int_{v} \nabla \cdot \mathbf{D}\, d\mathcal{V} = \oint_{S} \mathbf{D} \cdot d\mathbf{s}.$$

Gives integral form of Gauss's Law

$$\oint_{S} \mathbf{D} \cdot d\mathbf{s} = Q.$$

(integral form of Gauss's law)

Gauss's Law:

The integral form of Gauss's Law can be applied to determine **D** due to single isolated point charge q by enclosing the latter with a closed, spherical, Gaussian surface S of arbitrary radius R centered at c as shown in fig. From symmetry

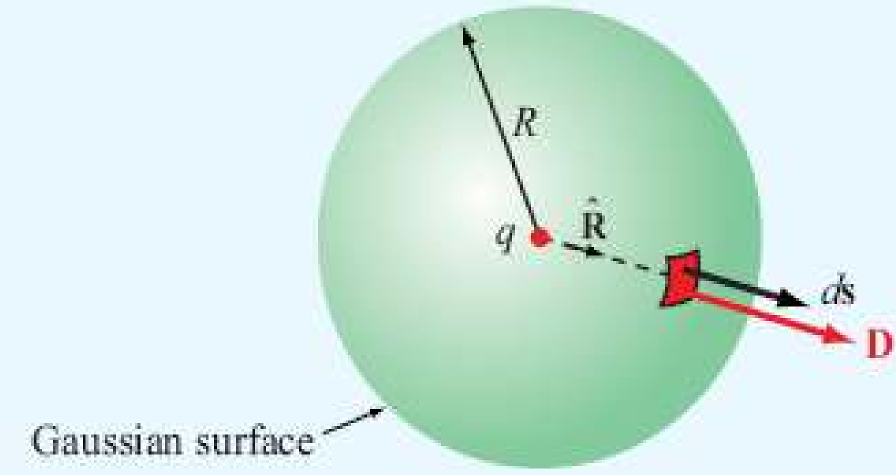

Figure 4-9 Electric field **D** due to point charge q.

Considerations and assuming that q is positive, the direction of **D** must be radially outward along the unit vector R, and D_R, the magnitude of D, must be the same at all point on S. Thus, at any point on S, $\mathbf{D} = \hat{\mathbf{R}} D_R$, and $d\mathbf{s} = \hat{\mathbf{R}}\, ds$. Applying Gauss's Law gives

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \oint_S \hat{\mathbf{R}} D_R \cdot \hat{\mathbf{R}}\, ds$$

$$= \oint_S D_R\, ds = D_R(4\pi R^2) = q.$$

Gauss's Law:

Solving for D_R and then inserting the results i $\mathbf{D} = \hat{\mathbf{R}} D_R,$ gives the following expression for the electric field E induced by an isolated point charge in a medium with permittivity ε.

$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon} = \hat{\mathbf{R}} \frac{q}{4\pi\epsilon R^2} \qquad \text{(V/m)}.$$

Even though Coulomb's law can be used to find **E** for any specified distribution of charge, Gauss's Law is easier to apply than Coulomb's Law, but its utility is limited to symmetrical charge distributions.

To successfully apply Gauss's Law, the surface S should be chosen such that, from symmetry considerations, across each subsurface of S, **D** is constant in magnitude and its direction is either normal or purely tangential to the subsurface.

Thank you !