University of Jeddah

Faculty of Engineering
Department of Electrical & Computer Engineering (ECE)

Control Systems (ECE 331)

Experiment No: 02

"Introduction to Simulink & Simulation of a Simple Speed Control System"

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Control Systems ECE 331

Experiment 02

Introduction to Simulink & Simulation of a Simple Speed Control System

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1 Introduction to Simulink

Simulink is a time based software package that is included in MATLAB and its main task is to solve Ordinary Differential Equations (ODEs) numerically. The need for the numerical solution comes from the fact that there is not an analytical solution for all Differential Equations, especially for those that are nonlinear.

The whole idea is to break the ODE into small time segments and to calculate the solution numerically for only a small segment. The length of each segment is called step size. Since the method is numerical and not analytical there will be an error in the solution. The error depends on the specific method and on the step size (usually denoted by h).

There are various formulas that can solve these equations numerically. Simulink uses Dormand-Prince (ODE5), fourth-order Runge-Kutta (ODE4), Bogacki-Shampine (ODE3), improved Euler (ODE2) and Euler (ODE1). A rule of thumb states that the error in ODE5 is proportional to h^5, in ODE4 to h4 and so on. Hence the higher the method the smaller the error.

Unfortunately the high order methods (like ODE5) are very slow. To overcome this problem variable step size solvers are used. When the systems states change very slowly then the step size can increase and hence the simulation is faster. On the other hand if the states change rapidly then the step size must be sufficiently small. The variable step size methods that Simulink uses are:

1. An explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair (ODE45).
2. An explicit Runge-Kutta (2,3) pair of Bogacki and Shampine (ODE23).
3. A variable-order Adams-Bashforth-Moulton PECE solver (ODE113).
4. A variable order solver based on the numerical differentiation formulas (NDFs) (ODE15s).
5. A modified Rosenbrock formula of order 2 (ODE23s).
6. An implementation of the trapezoidal rule using a "free" interpolant (ODE23t).
7. An implementation of TR-BDF2, an implicit Runge-Kutta formula with a first stage that is a trapezoidal rule step and a second stage that is a backward differentiation formula of order two (ODE23tb).

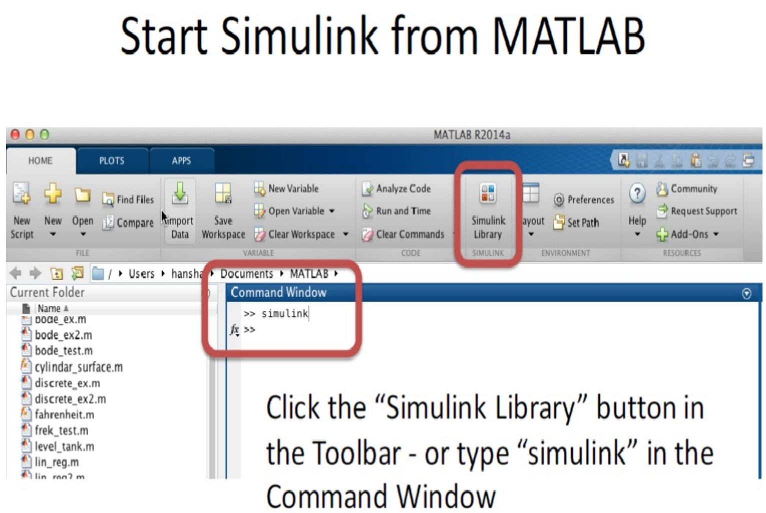

Figure 1: Front View of MATLAB Screen

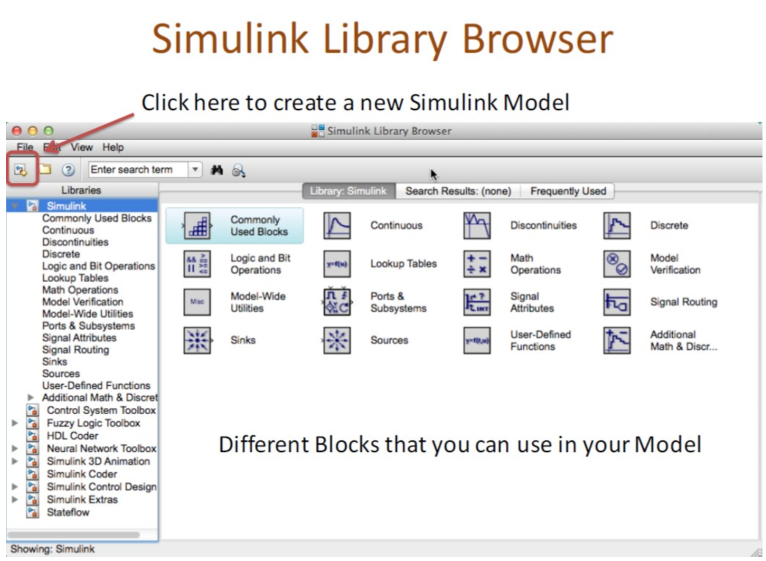

Figure 2: Simulink Library

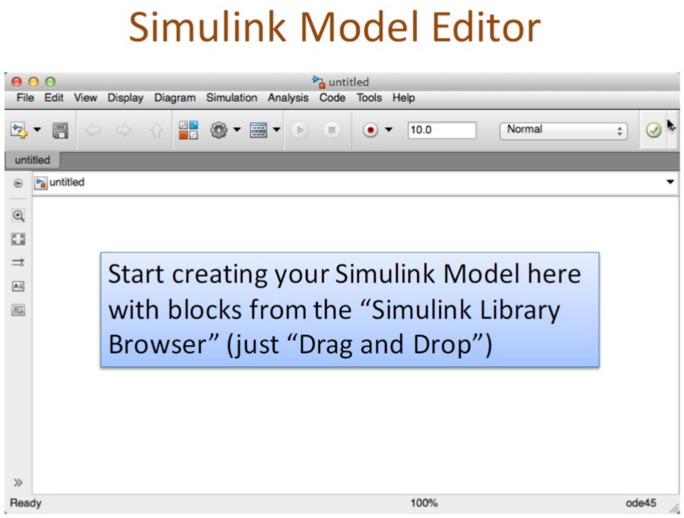

Figure 3: Simulink Model Editor

1.1 Example: 01 Generation of Sine waveform using Simulink.

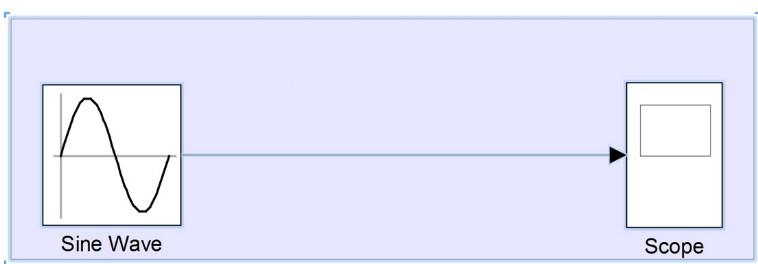

Figure 4: Simulink Model for Sine Wave Generation

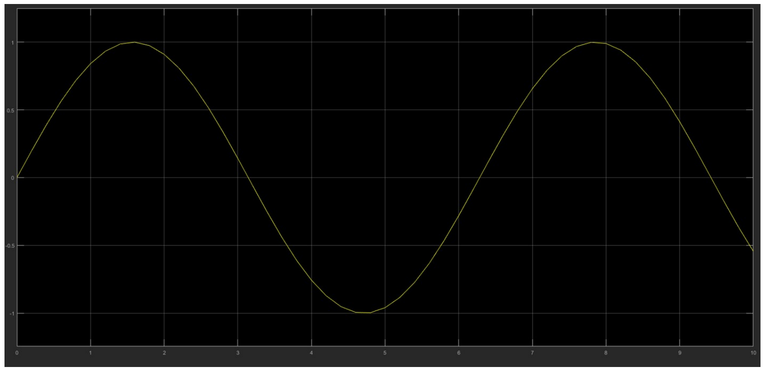

Figure 5: Generation of Sine Waveform using Simulink

1.2 Example: 02 Generation of Sine waveform, Transport Delay & Integrator using Simulink.

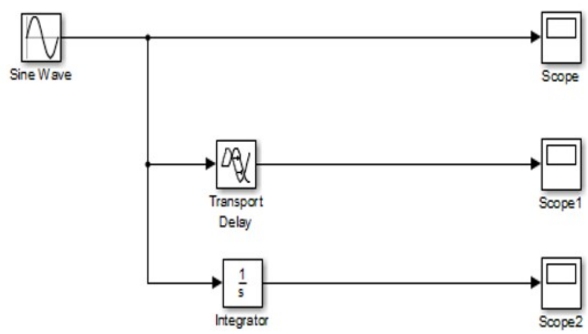

Figure 6: Simulink Model

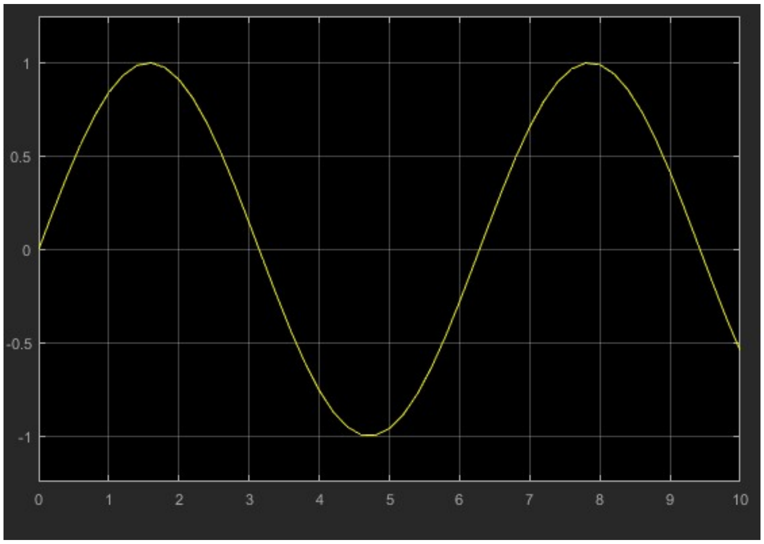

Figure 7: Sine Wave

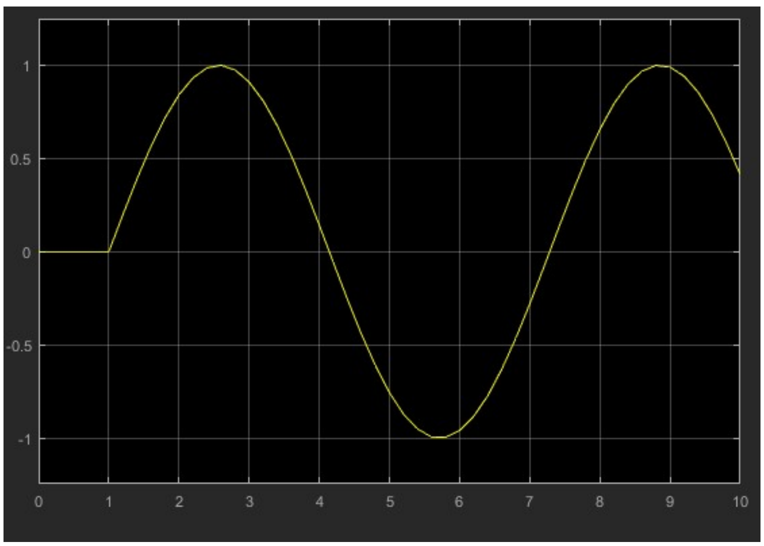

Figure 8: Transport Delay

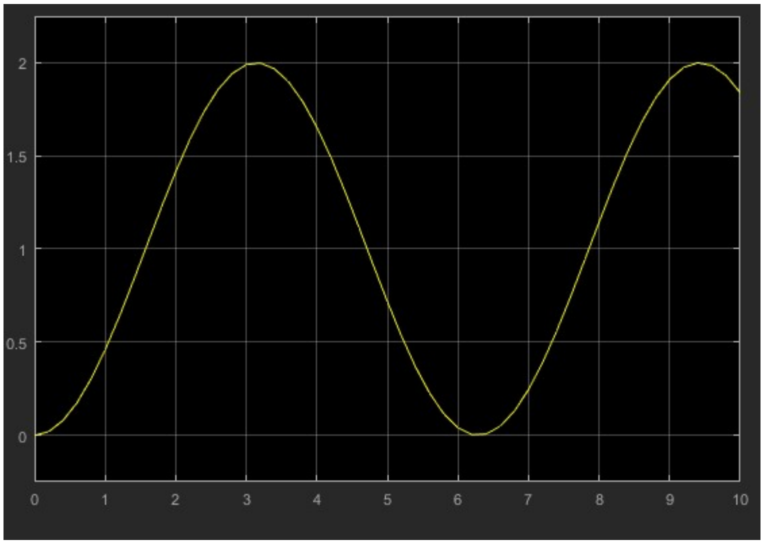

Figure 9: Integrator

1.3 Example: 03 Modeling of Closed Loop system using Simulink.

A simle model given as:

$\dot{x} = ax$

where $a = \frac{-1}{T}$

T is time constant

With this model, given conditions are also follow:

$T = 5$ $x(0) = 1$ $0 \le t \le 25$

Solution:

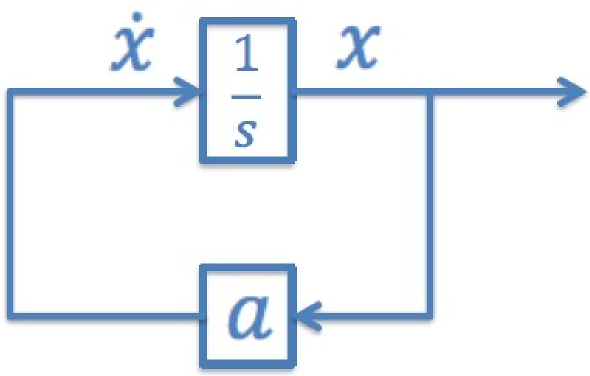

Figure 10: Model Information

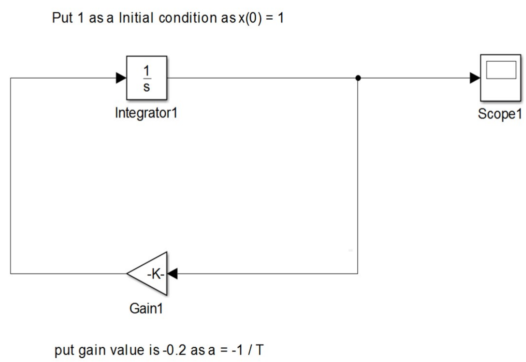

Figure 11: Modeling in Simulink

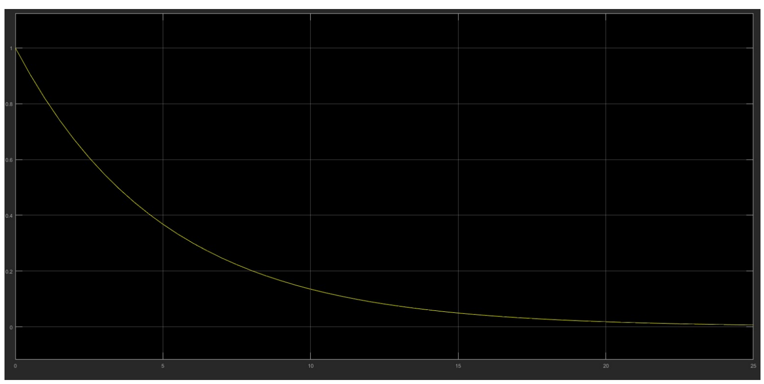

Figure 12: Result of the Model

2 Speed Control of Motor using Simulink

2.1 Physical Setup

A common actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion. The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in the following figure.

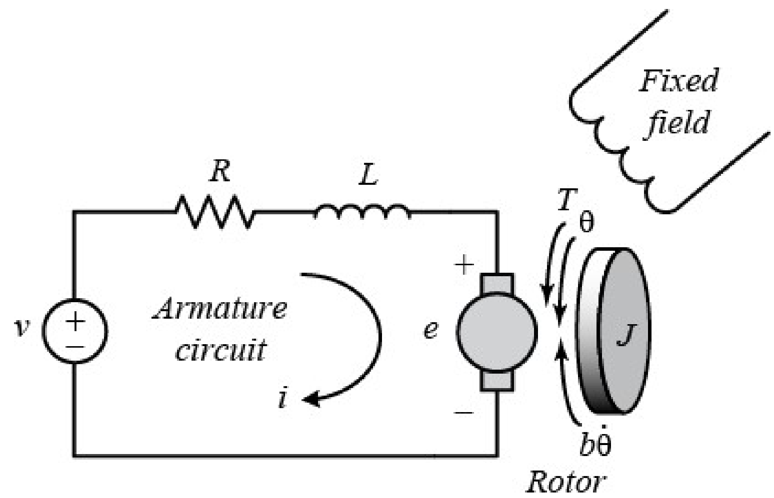

Figure 13: Physical Model of Motor

For this example, we will assume the following values for the physical parameters.

- Moment of Inertia of the Rotor (J) = $3.2284e^{-6}\ kg.m^2$
- Motor Viscous Friction Constant (b) = $3.5077e^{-6}\ N.m.s$
- Electromotive Force Constant (Kb) = $0.0274\ V/rad/sec$
- Motor Torque Constant (Kt) = $0.0274\ N.m/Amp$
- Electric Resistance (R) = $4\ Ohm$
- Electric Inductance (L) = $2.75e^{-6}\ H$

In this example, we assume that the input of the system is the voltage source (V) applied to the motor's armature, while the output is the position of the shaft (theta). The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity.

2.2 System Equations

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this example we will assume that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current i by a constant factor Kt as shown in the equation below. This is referred to as an armature-controlled motor.

$$T = K_t i$$

The back emf, e, is proportional to the angular velocity of the shaft by a constant factor Kb.

$$e = K_b \dot{\theta}$$

In SI units, the motor torque and back emf constants are equal, that is, $Kt = Ke$; therefore, we will use K to represent both the motor torque constant and the back emf constant.

From the figure above, we can derive the following governing equations based on Newton's 2nd law and Kirchhoff's voltage law.

$$J\ddot{\theta} + b\dot{\theta} = Ki$$
$$L\tfrac{di}{dt} + Ri = V - K\dot{\theta}$$

2.3 Transfer Function

Applying the Laplace transform, the above modeling equations can be expressed in terms of the Laplace variable s.

$$s(Js + b)\Theta(s) = KI(s)$$
$$(Ls + R)I(s) = V(s) - Ks\Theta(s)$$

We arrive at the following open-loop transfer function by eliminating $I(s)$ between the two above equations, where the rotational speed is considered the output and the armature voltage is considered the input.

$$P(s) = \tfrac{\dot{\Theta}(s)}{V(s)} = \tfrac{K}{(Js+b)(Ls+R)+K^2} \quad [\tfrac{rad/sec}{V}]$$

However, during this example we will be looking at the position as the output. We can obtain the position by integrating the speed, therefore, we just need to divide the above transfer function by s.

$$\tfrac{\Theta(s)}{V(s)} = \tfrac{K}{s((Js+b)+(Ls+R)+K^2)} \quad [\tfrac{rad}{V}]$$

Finally Transfer Function of the Motor is given by:

$$Transfer\ Function = \frac{0.0274}{8.878\ e^{-12}s^3+1.291\ e^{-5}s^2+0.0007648s}$$

This is a continuous time transfer function.

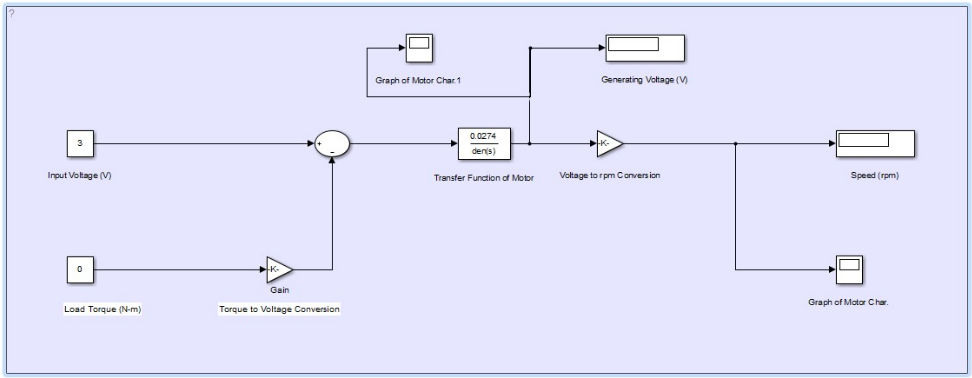

Figure 14: Simulink Model before running

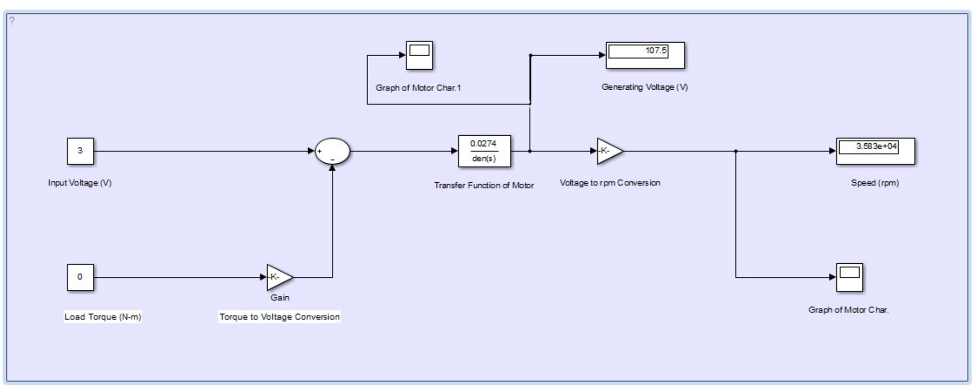

Figure 15: Simulink Model after running

The speed characteristics of Motor shown as under:

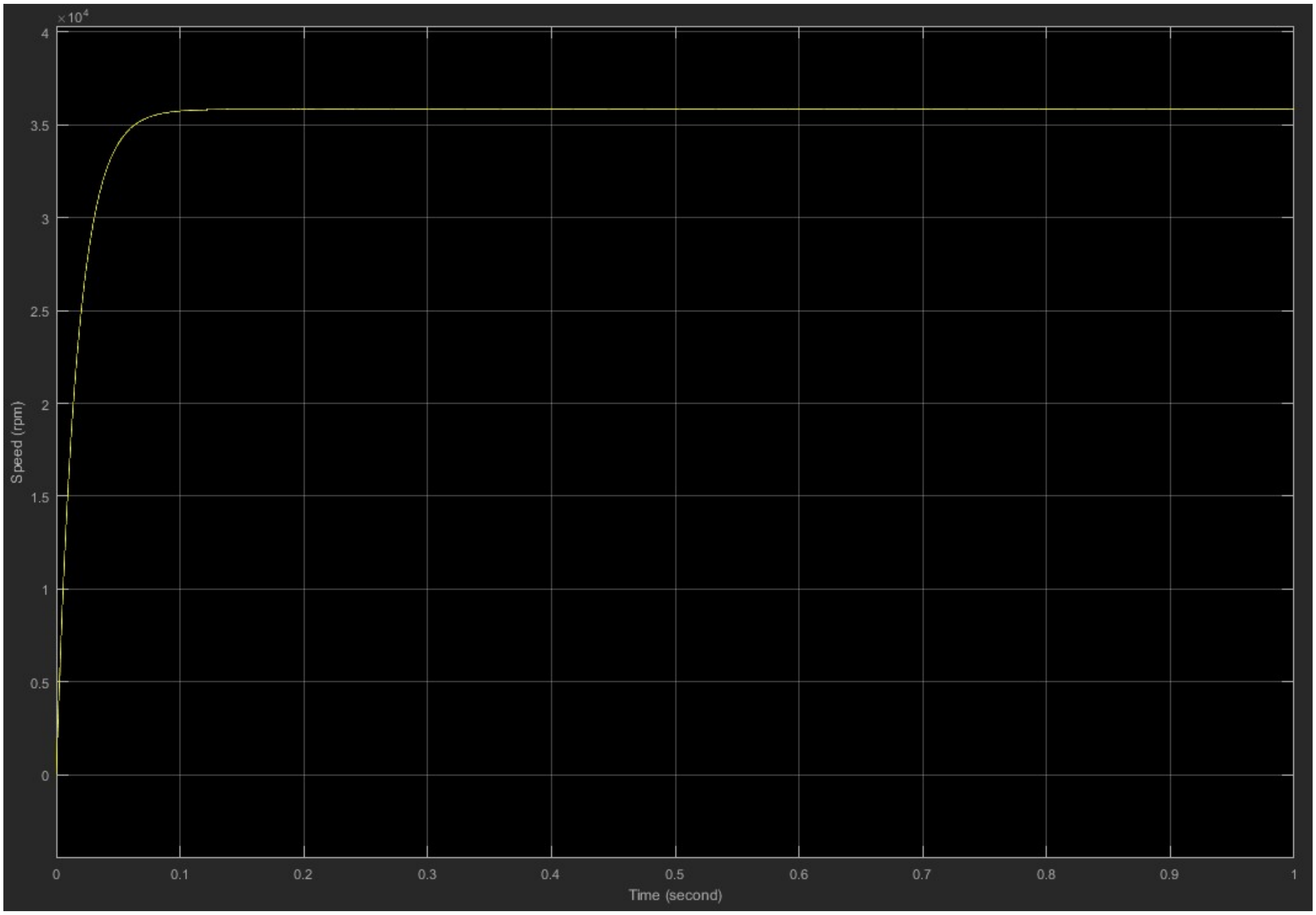

Figure 16: Speed Characteristics of Motor