جامعة جدة
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Control Systems (ECE 331)

Nonlinearity & Linearization

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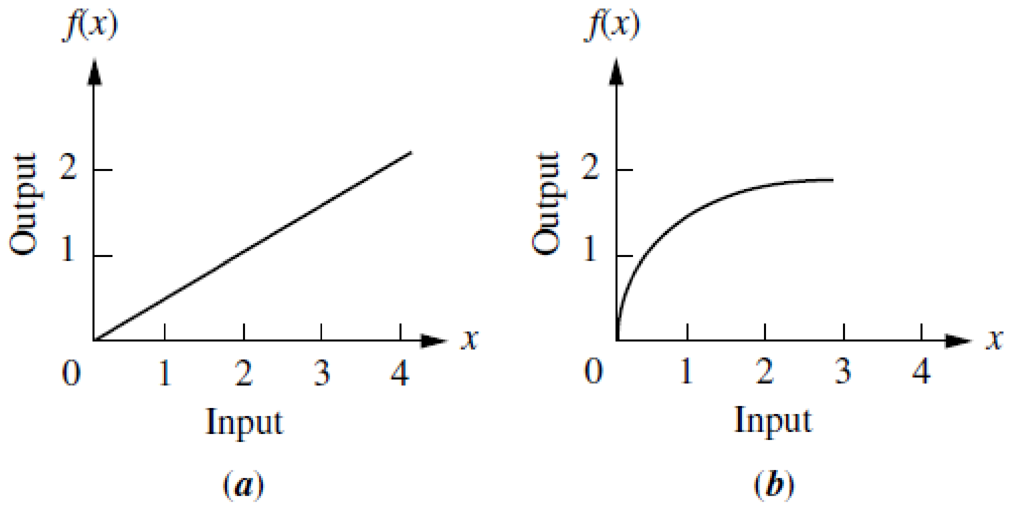
Nonlinearity:

The models are developed from systems are in form of approximately linear, time-invariant differential equations. An assumption of linearity was implicit in the development of these models.

A linear systems have two properties: Superposition and Homogeneity.

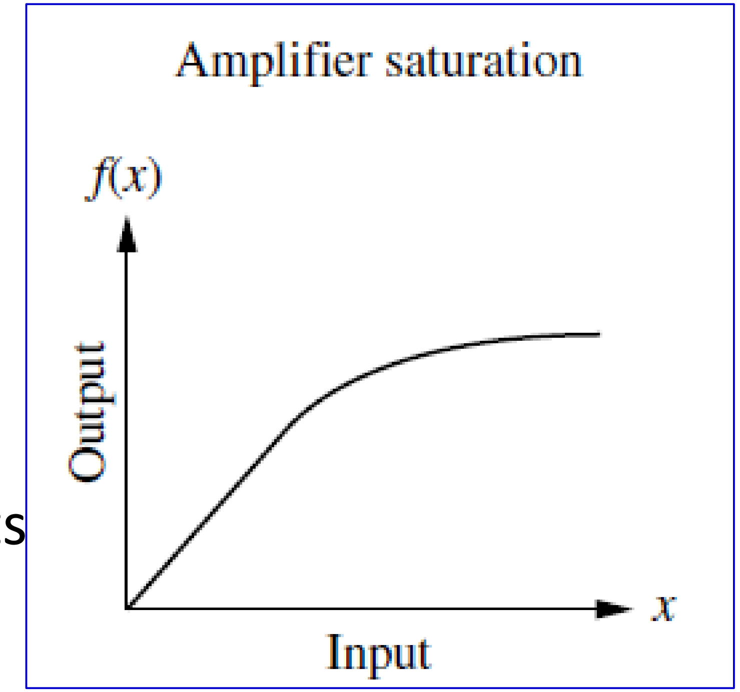
The property of Superposition means that the output response of a system to the sum of inputs is the sum of the responses to the individual inputs. Thus, if an input of $r_1(t)$ gives an output $c_1(t)$ and similarly $r_2(t)$ gives $c_2(t)$, then the input of $r_1(t)$+$r_2(t)$ gives an output of $c_1(t)$+$c_2(t)$.

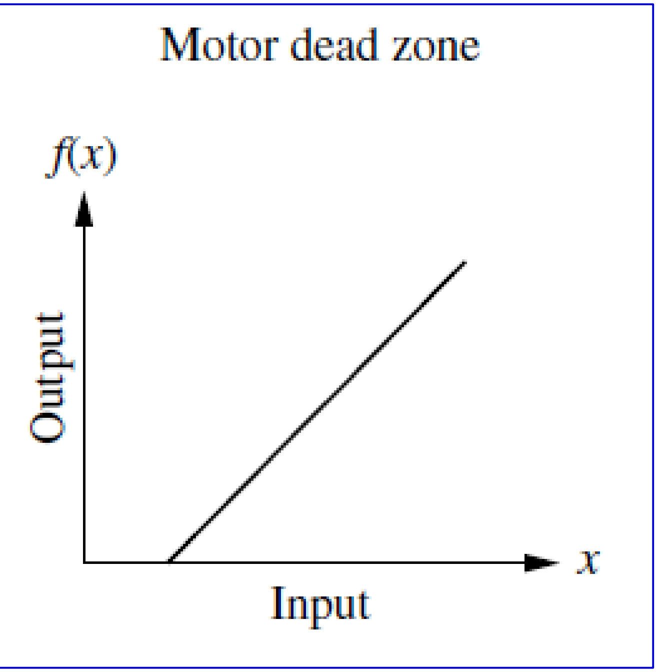


The property of Homogeneity describes the response of the system to a multiplication of the input by a scalar.
If $r_1(t)$ is input and $c_1(t)$ is output, then input of $Ar_1(t)$ gives output of $Ac_1(t)$ proven the concept of homogeneity.

Examples of Nonlinearities:

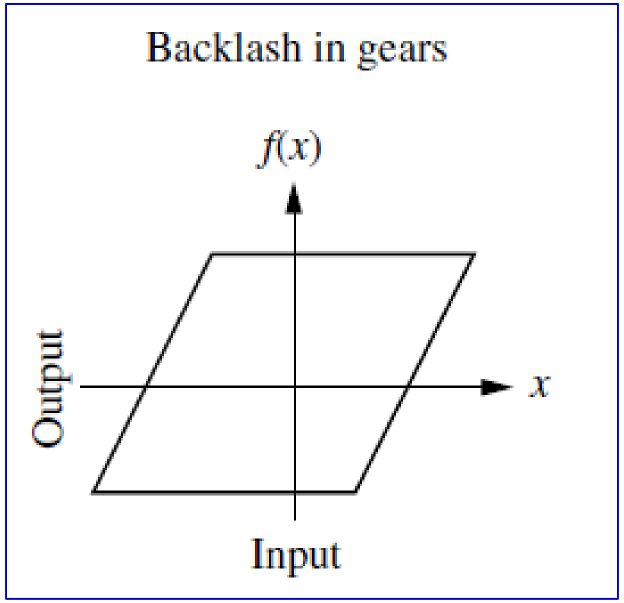
An electronic amplifier is linear over a specific range but exhibits the nonlinearity called "*Saturation*" at high input voltages.





A Motor that does not respond at very low input voltages due to frictional forces exhibits a nonlinearity called *"Dead Zone"*

Gears that do not fit tightly exhibit a nonlinearity called *"Backlash"* . The Input moves over a small range without the output responding.



Linearization:

If any nonlinear components are present, we must linearize the system before we can find the transfer function.

The first step is to recognize the nonlinear component and write a nonlinear differential equation. When we linearize a nonlinear differential equation, we linearize it for small-signal inputs about the steady-state solution when the small-signal input is equal to zero. This steady-state solution is called ”Equilibrium” and is selected as the second step in the linearization process.

For example: When the pendulum is at rest, it is at equilibrium. The angular displacement is described by a nonlinear differential equation, but it can be expressed with a linear differential equation for small excursions about this equilibrium point.

Secondly, we linearize the nonlinear differential equation, and then take the Laplace Transform of the linearized differential equation, assuming zero initial conditions.

Finally we separate input and output variables and form the transfer function.

Linearization:

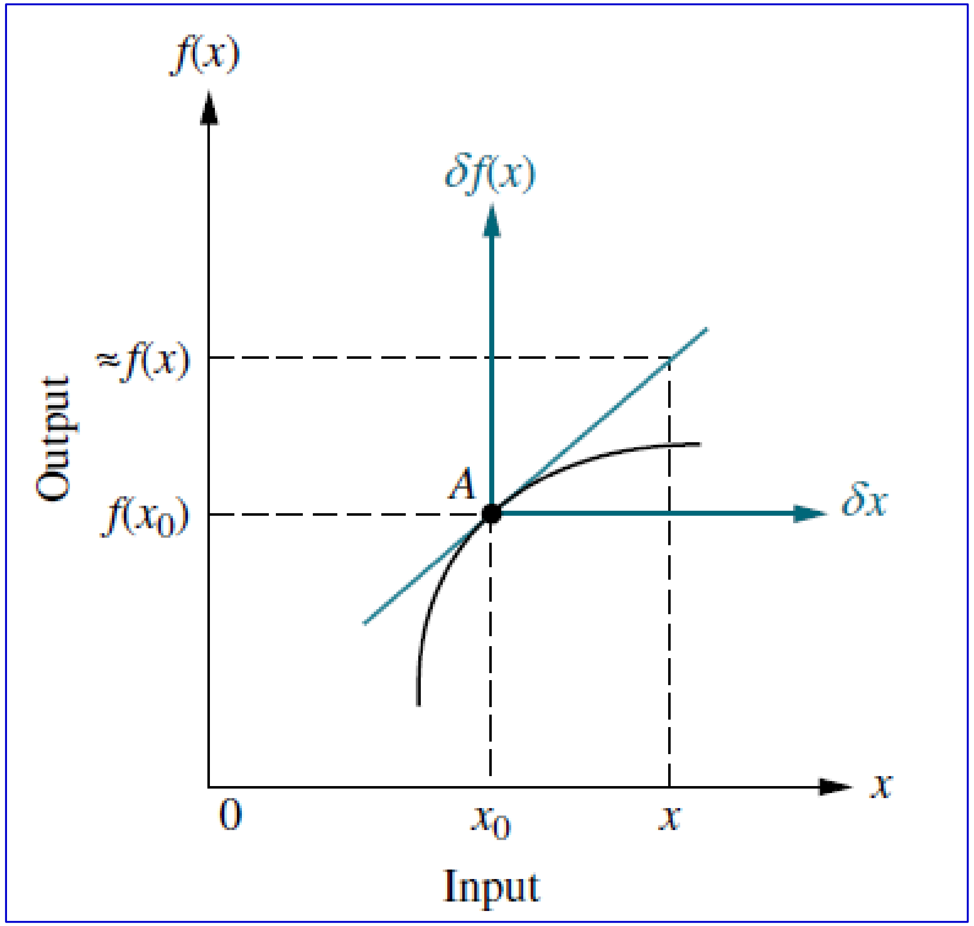
$$[f(x) - f(x_0)] \approx m_a(x - x_0)$$

from which

$$\delta f(x) \approx m_a \delta x$$

and

$$f(x) \approx f(x_0) + m_a(x - x_0) \approx f(x_0) + m_a \delta x$$



Thank You !