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Control Systems (ECE 331)

Steady State Errors

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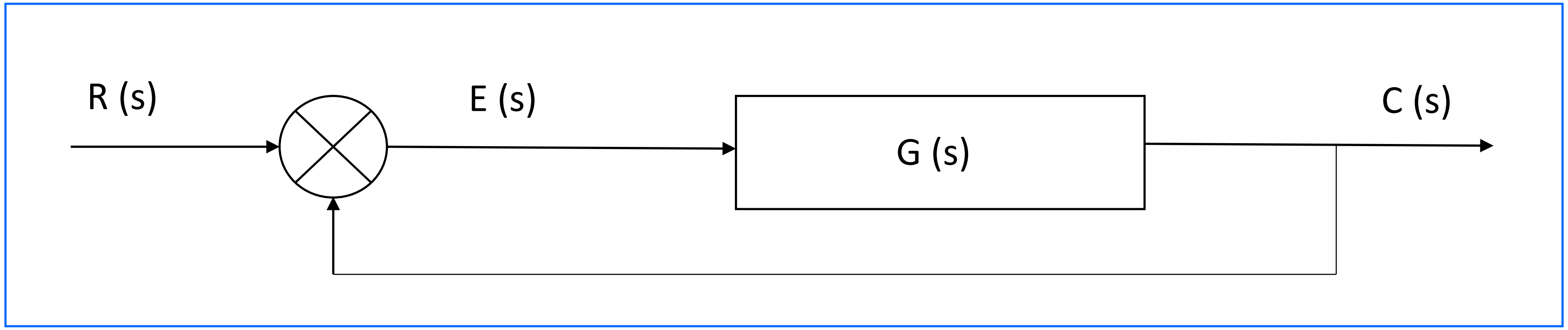
Steady State Errors in Control Theory :

The steady state error gives an idea to the system designer as to how accurate the designed system is. Steady state error may be caused in the system due to the nature of input of signal, type of system and non linearity present in the system. A system may show steady state error due to one type of input ad the same may be absent due to other inputs.

Consider a unity feedback system shown in fig.



Transfer function for the given system is:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

Steady State Errors in Control Theory : Conti…

But, $C(s) = E(s)G(s)$

So, the previous equation may write as

$$\frac{E(s)G(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\gg \; E(s) = \frac{R(s)}{1+G(s)}$$

- Steady state error is determined by application of final value theorem of Laplace Transform. Hence,

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$= \lim_{s\to 0} s\,E(s)$$

But, as $E(s) = \frac{R(s)}{1+G(s)}$

$$e_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{1+G(s)}$$

This shows that steady state error depends on the input and the forward path transfer function.

Type of Systems:

It is the way of designating of a control system and is difference from that of the order of a system. For open loop control system, transfer function is of the form,

$$G(s)H(s) = \frac{K(1 + sTa)(1 + sTb) + \cdots + (1 + sTm)}{s^n\,(1 + sT1)(1 + sT2) + \cdots + (1 + sTn)}$$

Where,

K=Gain

S^n = Represents pole of multiplicity *n* at the origin of the complex plane.

Remember::-- Type or number of system is governed by the number of integrations present in the open loop transfer function and is thus numerically equal to the number of poles of open loop transfer function at origin of the complex plane.

Type of Systems: Conti…

Type "0" System:

A system with "no integration" in the open loop transfer function <u>or</u> in the general form of open loop transfer function, n=0.

$$G(s)H(s) = \frac{K(s+1)\,\ldots.}{(s+2)(s+3)\,\ldots.}$$

Type "1" System:

A system with "one integration" in the open loop transfer function <u>or</u> in the general form of open loop transfer function, n=1.

$$G(s)H(s) = \frac{K(s+1)\,\ldots.}{s(s+2)(s+3)\,\ldots.}$$

Type "2" System:

A system with "two integration" in the open loop transfer function or in the general form of open loop transfer function, n=2.

$$G(s)H(s) = \frac{K(s+1)\,\ldots}{s^2(s+2)(s+3)\,\ldots}$$

Error Constants:

"Error constants or Coefficients are the measure of steady state errors and gives an idea as to how steady state error can be reduced or totally eliminated."

Position Error Constant:

Position error constant is defined for a unit step input. In the Laplace form unit step input is,

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} \frac{s\, R(s)}{1 + G(s)}$$

For unit step input, R(s) = 1/s, hence

$$e_{ss} = \lim_{s \to 0} \frac{s \times 1/s}{1+G(s)}$$ and hence $$e_{ss} = \lim_{s \to 0} \frac{1}{1+G(s)}$$

$$e_{ss} = \frac{1}{1 + K_p}$$

Where, Kp = Position Error Constant

Velocity Error Constant:

Velocity error constant is defined for a unit ramp input.

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \to 0} \frac{s\,R(s)}{1 + G(s)}$$

$$e_{ss} = \lim_{s \to 0} (s \times 1/s^2)/(1 + G(s))$$

and hence $e_{ss} = \lim_{s \to 0} \frac{1}{s + sG(s)}$

$$e_{ss} = \lim_{s \to 0} \frac{1}{sG(s)}$$

$$e_{ss} = \frac{1}{K_v}$$

Where, Kv = Velocity Error Constant

Error Constants: Conti…

Acceleration Error Constant:

Acceleration error constant is defined for a unit parabolic input.

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{1+G(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \times {}^{1}/_{s^3}}{1+G(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{1}{s^2 G(s)}$$

$$e_{ss} = \frac{1}{K_a}$$

Where, Ka = Acceleration Error Constant

Computation of Steady State Errors:

Note: Steady state error constants describe steady state errors when the inputs are three basic types: step, ramp and parabolic. For the other types of inputs no indication on steady state error is available.

Unit Step Input:

For unit step input the steady state error in terms of position constant is given by

$$e_{ss} = \frac{1}{1+K_p}$$

Type of System	Position Constant	Steady State Error
0	Kp	1/1+Kp
1	∞	0
2	∞	0
.	.	.
.	.	.
n, where n>0	∞	0

Computation of Steady State Errors: Conti...

Unit Ramp Input:

For unit ramp input the steady state error in terms of position constant is given by

$$e_{ss} = \frac{1}{K_v}$$

Type of System	Velocity Constant	Steady State Error
0	0	∞
1	Kv	Constant = 1/Kv
2	∞	0
.	.	.
.	.	.
n, where n>0	∞	0

Computation of Steady State Errors: Conti…

Unit Parabolic Input:

For unit parabolic input the steady state error in terms of position constant is given by

$$e_{ss} = \frac{1}{K_a}$$

Type of System	Acceleration Constant	Steady State Error
0	0	∞
1	0	∞
2	Ka	Constant = 1/Ka
3	∞	.
.	.	.
n, where n>0	∞	0

Computation of Steady State Errors: Conti…

It is very evident that error constants are either zero, finite or infinite. The magnitude of error constants will proportionally increase, if the inputs are greater than the unit value. Dimensionally, the error constants have the following units:

Position Error Constants: No Dimension

Velocity Error Constants: 1/Second

Acceleration Error Constants: 1/Second^2

Thank You !