University of Jeddah
Faculty of Engineering
Department of Electrical & Computer Engineering

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Electromagnetic Fields (ECE 308)

Lecture 9 – Electromagnetics

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Introduction:

Stationary charges produce static electric fields, and steady (i.e. non-time varying) currents produce static magnetic fields. When ∂/∂t = 0, the magnetic fields in a medium with magnetic permeability μ are governed by the second pair of Maxwell's eq.

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

J = Current Density

B = Magnetic Flux Density

H = Magnetic Field Intensity

Attribute	Electrostatics	Magnetostatics
Sources	Stationary charges ρ_v	Steady currents $\mathbf{J}$
Fields and fluxes	$\mathbf{E}$ and $\mathbf{D}$	$\mathbf{H}$ and $\mathbf{B}$
Constitutive parameter(s)	ϵ and σ	μ
Governing equations		
• Differential form	$\nabla \cdot \mathbf{D} = \rho_v$ $\nabla \times \mathbf{E} = 0$	$\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J}$
• Integral form	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$ $\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$ $\oint_C \mathbf{H} \cdot d\mathbf{l} = I$
Potential	Scalar V, with $\mathbf{E} = -\nabla V$	Vector $\mathbf{A}$, with $\mathbf{B} = \nabla \times \mathbf{A}$
Energy density	$w_e = \frac{1}{2}\epsilon E^2$	$w_m = \frac{1}{2}\mu H^2$
Force on charge q	$\mathbf{F}_e = q\mathbf{E}$	$\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$
Circuit element(s)	C and R	L

Magnetic Forces & Torques:

The electric field **E** at a point in space was defined as the electric force $\mathbf{F_e}$ per unit charge acting on a charged test particle placed at that point. We now define the magnetic flux density **B** at a point in space in terms of the magnetic force $\mathbf{F_m}$ that acts on a charged test particle moving with velocity **u** through that point. Experiments shows that a particle of charge q moving with velocity **u** in a magnetic field experiences a magnetic force $\mathbf{F_m}$ given by

$$\mathbf{F}_\mathrm{m} = q\mathbf{u} \times \mathbf{B}$$

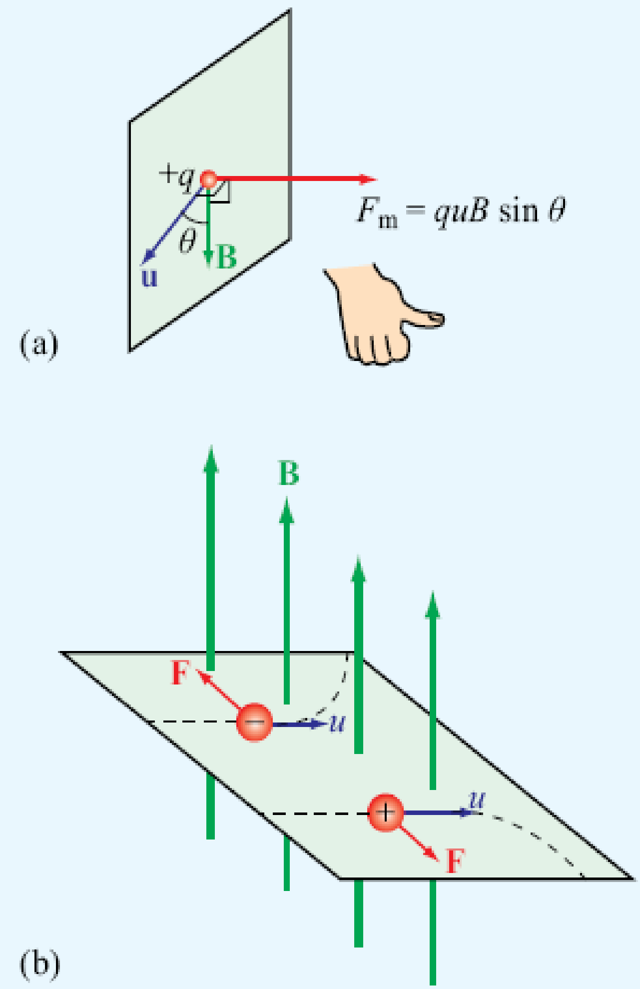

Figure 5-1 The direction of the magnetic force exerted on a charged particle moving in a magnetic field is (a) perpendicular to both **B** and **u** and (b) depends on the charge polarity (positive or negative).

Magnetic Forces & Torques:

For a positively charged particle, the direction of $\mathbf{F_m}$ is that of the cross product **u** × **B**, which is perpendicular to the plane containing **u** and **B** and governed by the **right hand rule**. If q is negative, the direction of $\mathbf{F_m}$ is reversed. The magnitude of $\mathbf{F_m}$ is given by : $\mathbf{F_m = q\ u\ B\ Sin\theta}$, where θ is the angle between **u** and **B**. Here note that $\mathbf{F_m}$ is maximum when u is perpendicular to B (θ=90°), and zero when u is parallel to B (θ=0° or 180°).

-If the charged particle resides in the presence of both an electric field **E** and a magnetic field **B**, then the total *Electromagnetic Force* acting on it is,

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_e + \mathbf{F}_m \\ &= q\mathbf{E} + q\mathbf{u} \times \mathbf{B} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \end{aligned}$$

This is also known as ***"Lorentz Force"***

Magnetic Forces & Torques:

Difference between electric and magnetic forces:

1. Whereas the electric force is always in the direction of the electric field, the magnetic force is always perpendicular to the magnetic field.
2. Whereas the electric force acts on a changed particle whether or not it is moving, the magnetic force acts on it only when it is in motion.
3. Whereas the electric force expends energy in displacing a charged particle, the magnetic force does no work when a particle is displaced. (Because magnetic force Fm is always perpendicular to u, Fm · u = 0. Hence, the work performed when a particle is displaced by a differential distance : dl = u dt is $dW = \mathbf{F}_m \cdot d\mathbf{l} = (\mathbf{F}_m \cdot \mathbf{u})\, dt = 0.$

Since no work is done, a magnetic field cannot change the K.E. of a charged particle; the magnetic field can change the direction of motion of a charged particle, not its speed.

The Biot-Savart Law:

Through his experiments on the deflection of compass needles by current-carrying wires, Oersted established that currents induce magnetic fields that form closed loops around the wire. Building upon Oersted's results, Biot and Savart arrived at an expression that relates the magnetic field **H** at any point in

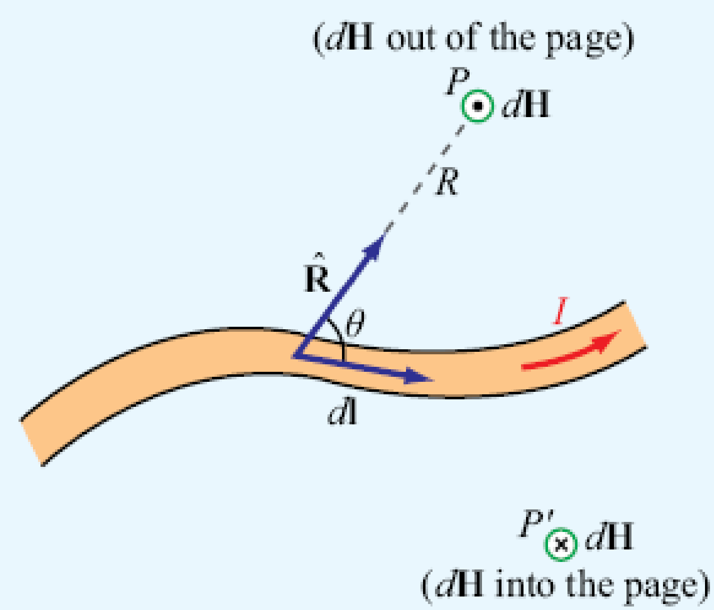

Figure 5-8 Magnetic field $d\mathbf{H}$ generated by a current element $I\,d\mathbf{l}$. The direction of the field induced at point P is opposite to that induced at point P'.

space to the current I that generates **H**. The Biot-Savart states that **"the differential magnetic field *d*H generated by a steady current I flowing through a differential path length vector *d*l is**

$$d\mathbf{H} = \frac{I}{4\pi}\frac{d\mathbf{l}\times\hat{\mathbf{R}}}{R^2}$$

Gauss Law for Magnetism:

Till time, we have studied Biot-Savart law for finding the magnetic flux density B and field H due to any distribution of electric currents in free space, and we examined how magnetic fields can exert magnetic forces on moving charged particles and current-carrying conductors. This is by

i) Gauss Law, and ii) Ampere's Law

Gauss Law: We know that the net outward flux of the electric flux density D through a closed surface equals the enclosed net charge Q, This is known as Gauss Law for Electrostatics.

$$\nabla \cdot \mathbf{D} = \rho_v \quad \longleftrightarrow \quad \oint_S \mathbf{D} \cdot d\mathbf{s} = Q.$$

Conversion from differential to integral form was accomplished by applying divergence theorem:

$$Q = \int_V \rho_v \, dV$$

The Magnetostatics counterpart is given by:

$$\nabla \cdot \mathbf{B} = 0 \quad \longleftrightarrow \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0.$$

This is known as Gauss Law for Magnetics.

Gauss Law for Magnetism:

The differential form is one of Maxwell's four equations, and the integral form is obtained with help of the divergence theorem. Note that the right hand side of Gauss's law is zero, It shows that the magnetic equivalence of an electric point charge does not exists in nature.

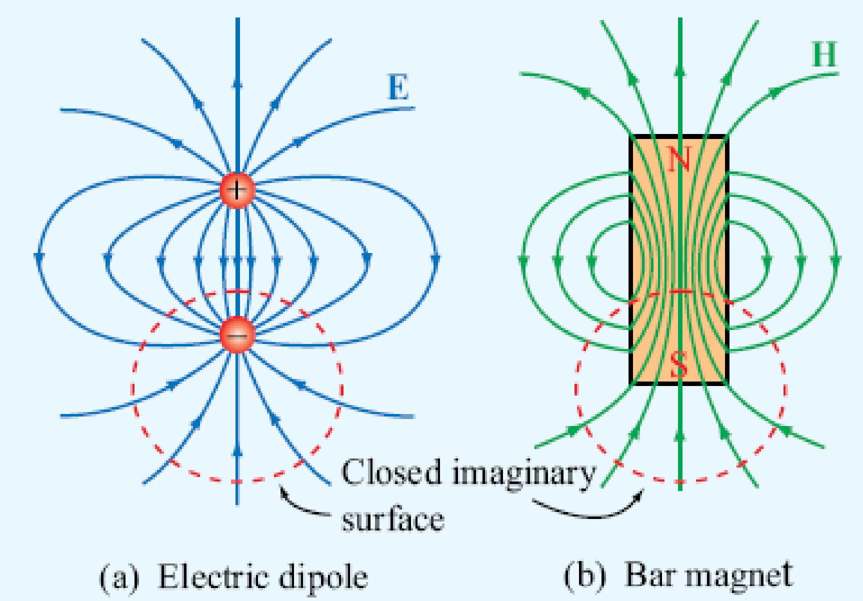

Figure 5-15 Whereas (a) the net electric flux through a closed surface surrounding a charge is not zero, (b) the net magnetic flux through a closed surface surrounding one of the poles of a magnet is zero.

-The difference between Gauss's law for electric and magnetic is defined by its field lines. In electric, field lines originate from positive electric charges and terminate on negative ones. In magnetic, field lines always form continuous closed loops, as shown in fig.

Ampere's Law for Magnetism:

In electrostatics, line integral along a closed contour always vanishes.

$$\nabla \times \mathbf{E} = 0 \quad \Longleftrightarrow \quad \oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = 0.$$

Conversion of differential to integral form was accomplished by applying Stokes's theorem to a surface S with contour C.

The Magnetostatics counterpart is known as **Ampere's Law.**

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \Longleftrightarrow \quad \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I$$

"Ampere's circuital law states that the line integral of H around a closed path is equal to the current traversing the surface bounded by that path".

Thank you !