Faculty of Engineering
Department of Electrical & Computer Engineering

Control Systems (ECE 331)

Mathematical Modeling of Physical Systems

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Introduction to Modeling:

"Mathematical Modeling is the process of constructing mathematical objects, whose behaviors or properties correspond in some way to a particular real-world system".

- A mathematical object could be a system of equations, a stochastic process, a geometric or algebraic structure, an algorithm, or even just a set of numbers.

- The term real world system could refer to a physical system, a financial system, a social system, an economical system, or essentially any other system whose behaviors can be observed.

Why Model?:

There are many reasons, but out of all there are two main reasons as under:

- To gain understanding: Generally speaking, if we have a mathematical model which accurately reflects some behavior of a real-world system of interest, we can often gain improved understanding of that system through analysis of the model. Furthermore, in the process of building the model we find out which factors are most important in the system, and how different parts of the system are related.

- To predict or simulate: Very often we wish to know what a real-world system will do in the future, but it is expensive, impractical, or impossible to experiment directly with the system. Examples include nuclear reactor design, space flight, weather prediction, drug efficiency in human, and so on.

The Modeling Process:

Unfortunately, there is no definite “algorithm” to construct a mathematical model that will work in all situations. Modeling is sometimes viewed as an art. It involves taking whatever knowledge you may have of mathematics and of the system of interest and using that knowledge to create something.

Since everyone has a different knowledge base, a preferred bag of tricks, and a unique way of looking at problems, different people may come up with different models for the same system.

There is usually plenty of room for argument about which model is ‘best’.

The Modeling Process: conti...

It is very important to understand at the outset that for any real system, there is no "perfect" model. One is always faced with tradeoffs between:

- Accuracy
- Flexibility
- Cost

Increasing the accuracy of a model generally increases cost and decreases flexibility. The goal in creating a model is usually to obtain a "sufficiently accurate" and flexible model at a low cost.

The Modeling Process: conti…

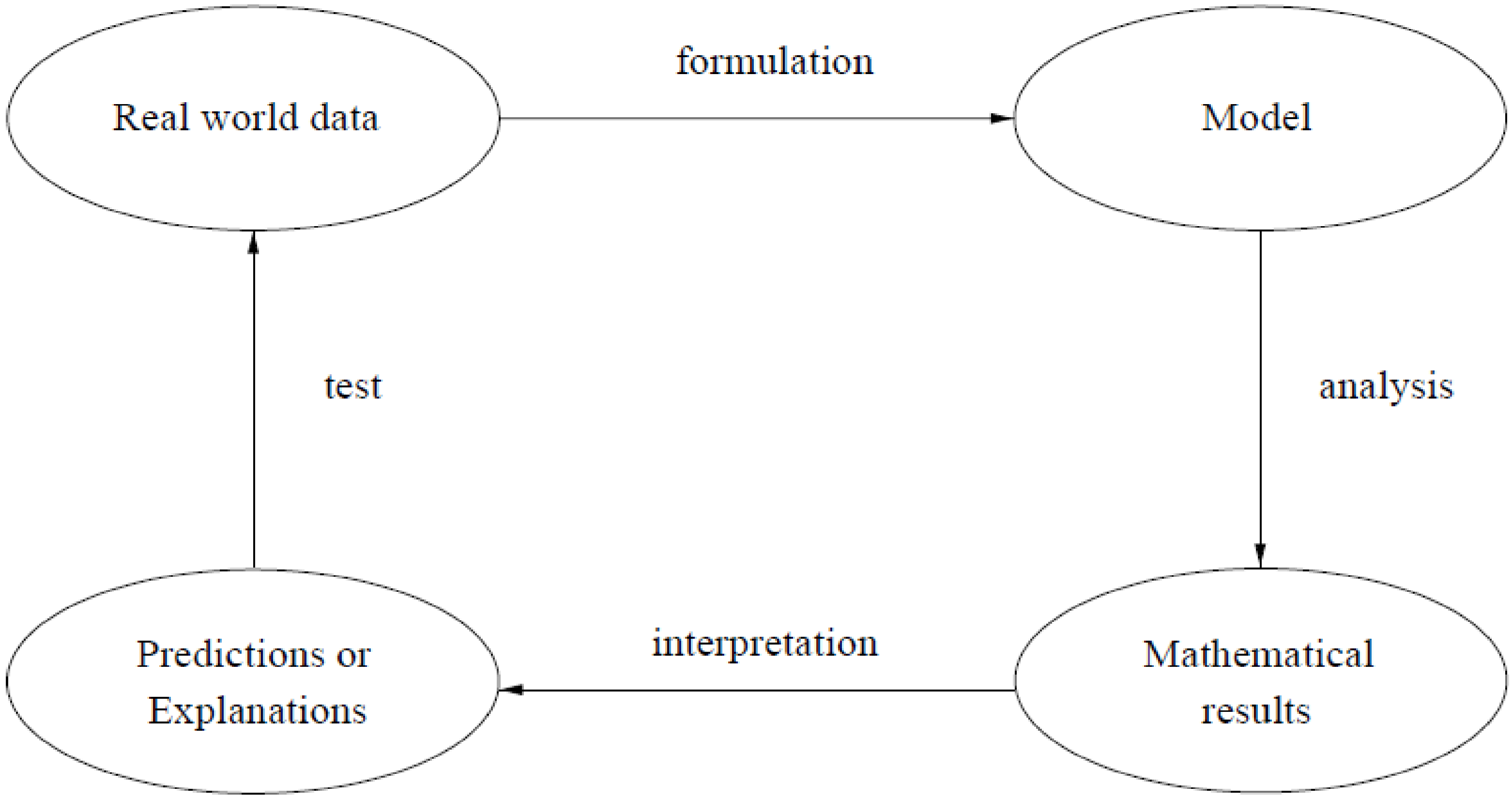

Figure 1: Schematic flow of modeling process.

The Modeling Process: conti...

Constructing a model requires:

- A clear picture of the goal of the modeling exercise. Exactly which aspects of the system do you wish to understand or predict, and how accurately do you need to do it?
- An picture of the key factors involved in the system and how they relate to each other. This often requires taking a greatly simplified view of the system, neglecting factors known to influence the system, and making assumptions which may or may not be correct.

Linear Systems:

A system is called linear if the principle of superposition applies.

"The principle of superposition states that the response produced by the simultaneous application of two different forcing functions in the sum of the two individual responses".

Hence, for the linear system, the response to several inputs can be calculated by treating one input at a time and adding the results. It is this principle that allows one to build up complicated solutions to the linear differential equation from simple solutions.

Introduction to Modeling: conti…

Linear Time Invariant Systems & Time Varying Systems:

"Dynamical systems that are composed of linear time invariant lumped parameter components may be described by linear time-invariant differential equations-that is , constant co-efficient differential equations. Such systems are called Linear Time Invariant Systems".

"Systems that are represented by differential equations whose coefficients are functions of time are called Linear Time Varying Systems."

Example: The mass of a spacecraft changes due to fuel consumption.

Thank You !