University of Jeddah
Faculty of Engineering
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Electromagnetic Fields (ECE 308)

Lecture 1 – Vector Analysis - I

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What is Electromagnetic Field?

An electromagnetic field is a physical field produced by electrically charged objects. It affects the behavior of charged objects in the vicinity of the field. The electromagnetic field extends indefinitely throughout space and describes the electromagnetic interaction.

The field can be viewed as the combination of an electric field and a magnetic field. The electric field is produced by stationary charges, and the magnetic field by moving charges (currents); these two are often described as the sources of the field.

Vector Analysis:

In science and engineering we frequently encounter quantities that have magnitude and magnitude only: mass, time, temperature, etc. These are known as **Scalar** quantities, which remains the same no matter what coordinates we use.

The second group includes displacement, velocity, acceleration, force, momentum and angular momentum. Quantities with magnitude and direction are known as **Vector** quantities.

Generally, to distinguish vectors from scalars, we identify vector quantities with boldface type, that is **V.**

Laws of Vector Algebra:

A vector is a mathematical object that resembles an arrow. Vector **A** in fig. has magnitude (or length) A = |A| and a unit vector

$$\mathbf{A} = \hat{\mathbf{a}}|\mathbf{A}| = \hat{\mathbf{a}}A.$$

$$\hat{\mathbf{a}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}.$$

$$\mathbf{A} = \hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z,$$

$$A = |\mathbf{A}| = \sqrt[+]{A_x^2 + A_y^2 + A_z^2}.$$

$$\hat{\mathbf{a}} = \frac{\mathbf{A}}{A} = \frac{\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z}{\sqrt[+]{A_x^2 + A_y^2 + A_z^2}}.$$

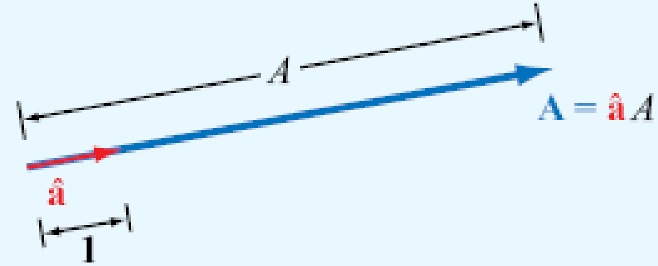

Figure 3-1 Vector $\mathbf{A} = \hat{\mathbf{a}}A$ has magnitude $A = |\mathbf{A}|$ and points in the direction of unit vector $\hat{\mathbf{a}} = \mathbf{A}/A$.

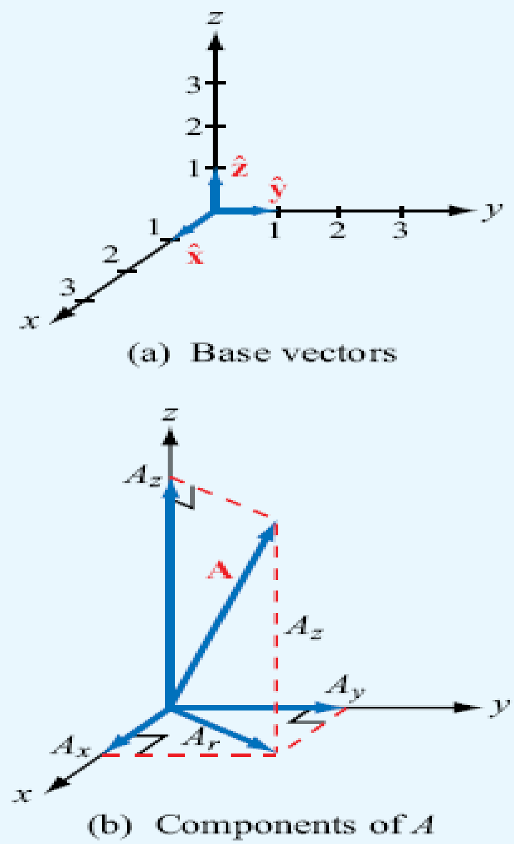

Figure 3-2 Cartesian coordinate system: (a) base vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$, and (b) components of vector **A**.

Laws of Vector Algebra:

Equality of Two Vectors:

Two vectors **A** and **B** are equal if they have equal magnitudes and identical unit vectors. Thus, if

$$\mathbf{A} = \hat{\mathbf{a}}A = \hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z,$$

$$\mathbf{B} = \hat{\mathbf{b}}B = \hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z,$$

Then A = B if and only if A = B and their unit vectors are equals, which requires that

$$A_x = B_x, \quad A_y = B_y, \text{ and } A_z = B_z$$

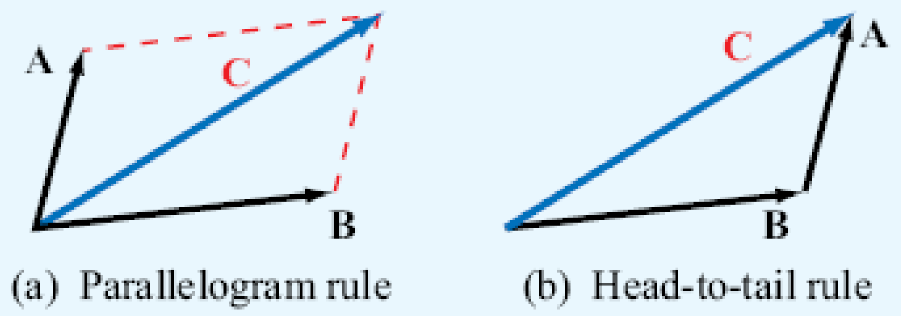

Figure 3-3 Vector addition by (a) the parallelogram rule and (b) the head-to-tail rule.

Note: Equality of two vectors does not necessarily imply that they are identical; in Cartesian coordinates, two displaced parallel vectors of equal magnitude and pointing in the same direction are equal, but they are identical only if they lie on top of one another.

Laws of Vector Algebra:

Vector Addition & Subtraction:

The sum of two vectors **A** and **B** is a vector $\mathbf{C} = \hat{\mathbf{x}} C_x + \hat{\mathbf{y}} C_y + \hat{\mathbf{z}} C_z$ is given by

$$\begin{aligned}\mathbf{C} &= \mathbf{A} + \mathbf{B} \\ &= (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z) + (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z) \\ &= \hat{\mathbf{x}}(A_x + B_x) + \hat{\mathbf{y}}(A_y + B_y) + \hat{\mathbf{z}}(A_z + B_z) \\ &= \hat{\mathbf{x}} C_x + \hat{\mathbf{y}} C_y + \hat{\mathbf{z}} C_z.\end{aligned}$$

Hence the vector addition is:

$$\mathbf{C} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}.$$

The vector subtraction is:

$$\begin{aligned}\mathbf{D} &= \mathbf{A} - \mathbf{B} \\ &= \mathbf{A} + (-\mathbf{B}) \\ &= \hat{\mathbf{x}}(A_x - B_x) + \hat{\mathbf{y}}(A_y - B_y) + \hat{\mathbf{z}}(A_z - B_z)\end{aligned}$$

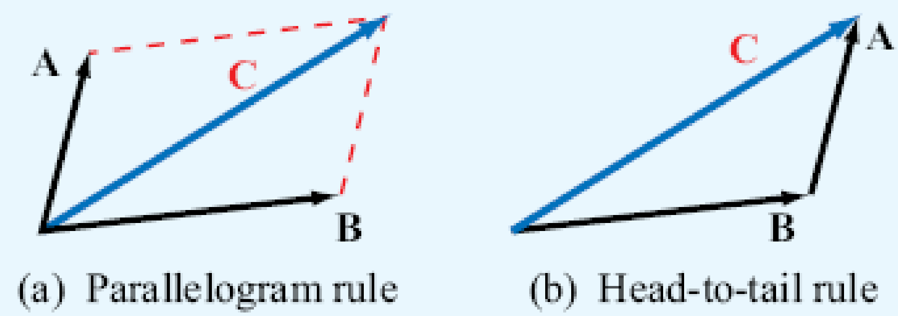

Figure 3-3 Vector addition by (a) the parallelogram rule and (b) the head-to-tail rule.

Laws of Vector Algebra:

Position and Distance Vector:

The **Position Vector** of a point P in space is the vector from the origin of P. Assuming points P_1 and P_2 are at (x_1,y_1,z_1) and (x_2,y_2,z_2) as in fig. , their position vectors are:

$$\mathbf{R}_1 = \overrightarrow{OP_1} = \hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1,$$

$$\mathbf{R}_2 = \overrightarrow{OP_2} = \hat{\mathbf{x}}x_2 + \hat{\mathbf{y}}y_2 + \hat{\mathbf{z}}z_2,$$

Where point O is the origin. The **Distance Vector** from P_1 to P_2 is defined as

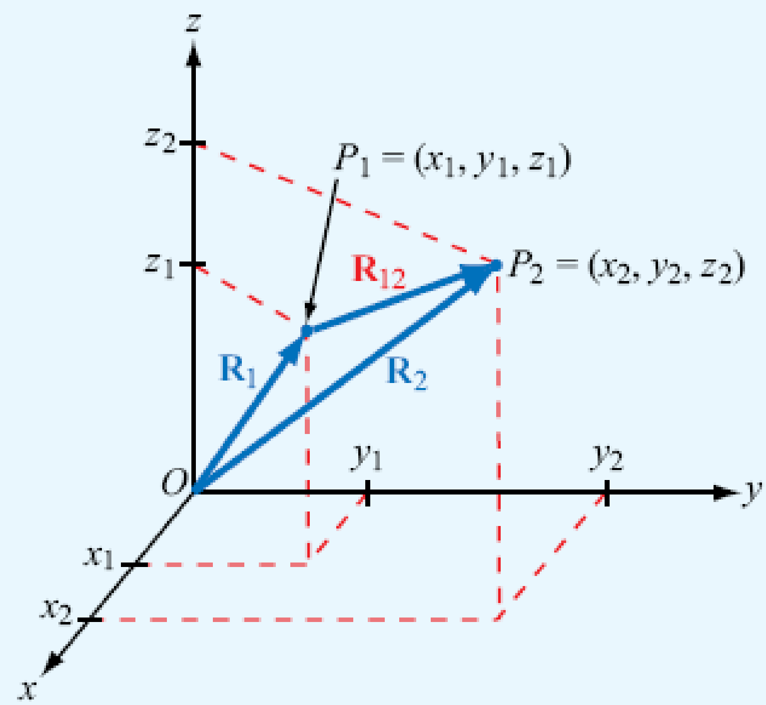

Figure 3-4 Distance vector $\mathbf{R}_{12} = \overrightarrow{P_1P_2} = \mathbf{R}_2 - \mathbf{R}_1$, where $\mathbf{R}_1$ and $\mathbf{R}_2$ are the position vectors of points P_1 and P_2, respectively.

$$\begin{aligned}\mathbf{R}_{12} &= \overrightarrow{P_1P_2} \\ &= \mathbf{R}_2 - \mathbf{R}_1 \\ &= \hat{\mathbf{x}}(x_2 - x_1) + \hat{\mathbf{y}}(y_2 - y_1) + \hat{\mathbf{z}}(z_2 - z_1),\end{aligned}$$

$$\begin{aligned}d &= |\mathbf{R}_{12}| \\ &= [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{1/2}.\end{aligned}$$

Laws of Vector Algebra:

Vector Multiplication:

[1] Simple Product: The multiplication of a vector by scalar is called Simple Product.

$$\mathbf{B} = k\mathbf{A} = \hat{\mathbf{a}}kA = \hat{\mathbf{x}}(kA_x) + \hat{\mathbf{y}}(kA_y) + \hat{\mathbf{z}}(kA_z)$$
$$= \hat{\mathbf{x}}\,B_x + \hat{\mathbf{y}}\,B_y + \hat{\mathbf{z}}\,B_z.$$

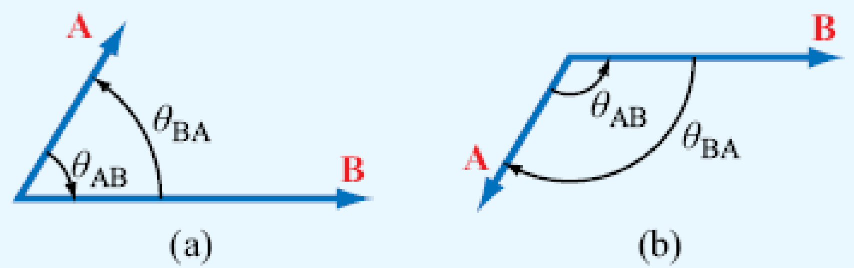

Figure 3-5 The angle θ_{AB} is the angle between **A** and **B**, measured from **A** to **B** between vector tails. The dot product is positive if $0 \le \theta_{AB} < 90°$, as in (a), and it is negative if $90° < \theta_{AB} \le 180°$, as in (b).

[2] Scalar or Dot Product: Dot product of two co-anchored vectors **A** and **B,** denoted **A · B**, is defined geometrically as the product of the magnitude of **A** and the scalar component of **B** along **A**, or vice versa.

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta_{AB},$$

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A},$$

(commutative property)

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C},$$

(distributive property)

Laws of Vector Algebra:

Vector Multiplication:

[3] Vector or Cross Product: The vector product of two vectors given by:

$$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}}\, AB \sin\theta_{AB},$$

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \quad \text{(anticommutative)}.$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C},$$

(distributive)

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}.$$

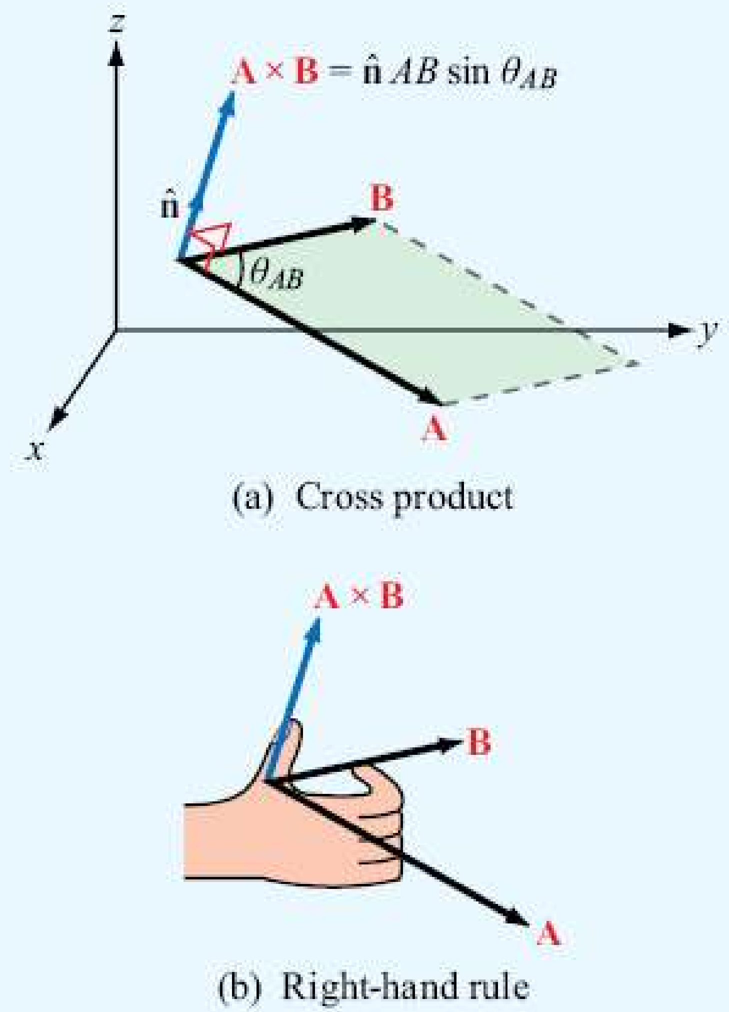

Figure 3-6 Cross product $\mathbf{A} \times \mathbf{B}$ points in the direction $\hat{\mathbf{n}}$, which is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ and defined by the right-hand rule.

Thank you !