Faculty of Engineering
Department of Electrical & Computer Engineering

Control Systems (ECE 331)

Signal Flow Graph (SFG) - I

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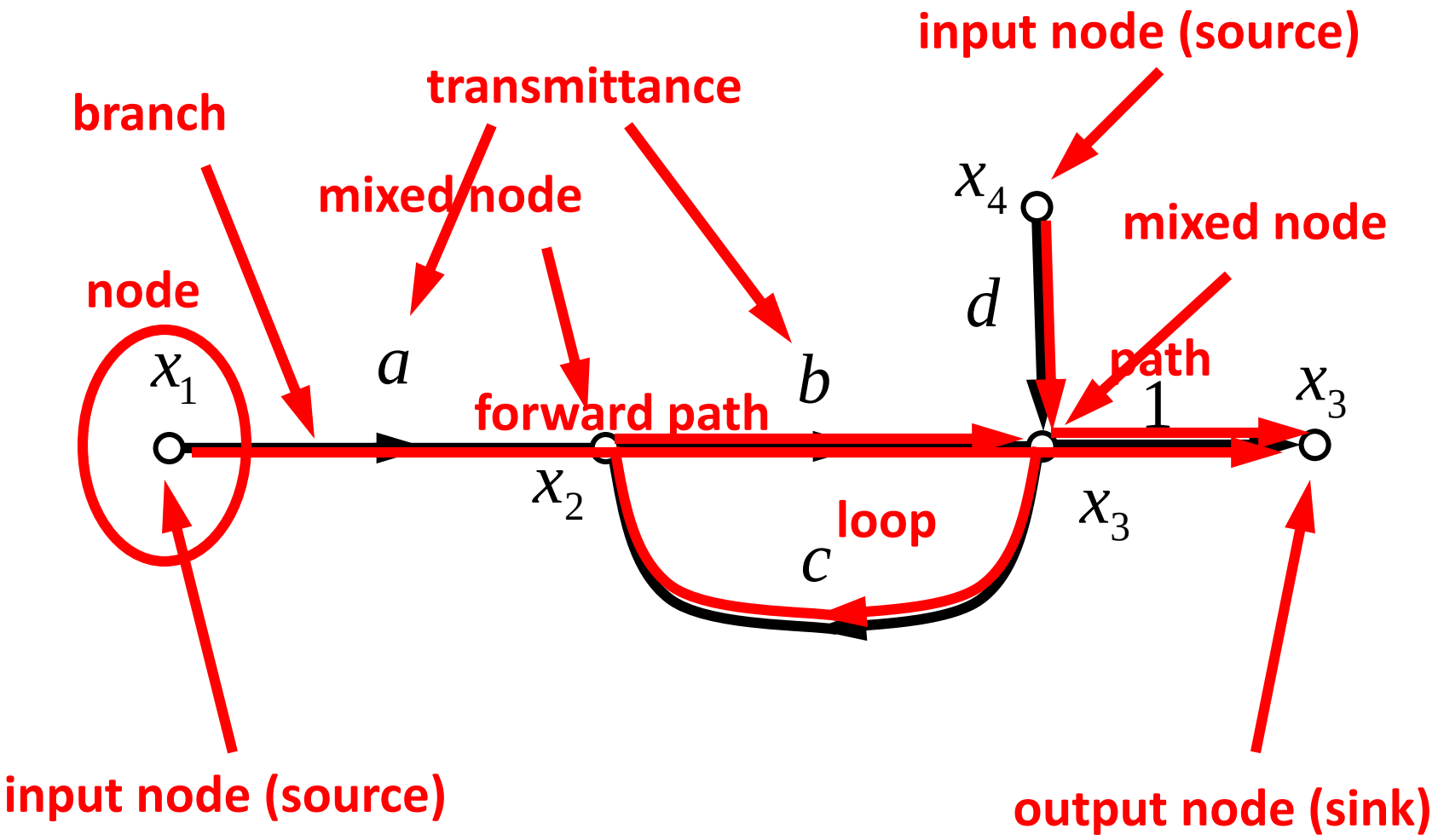
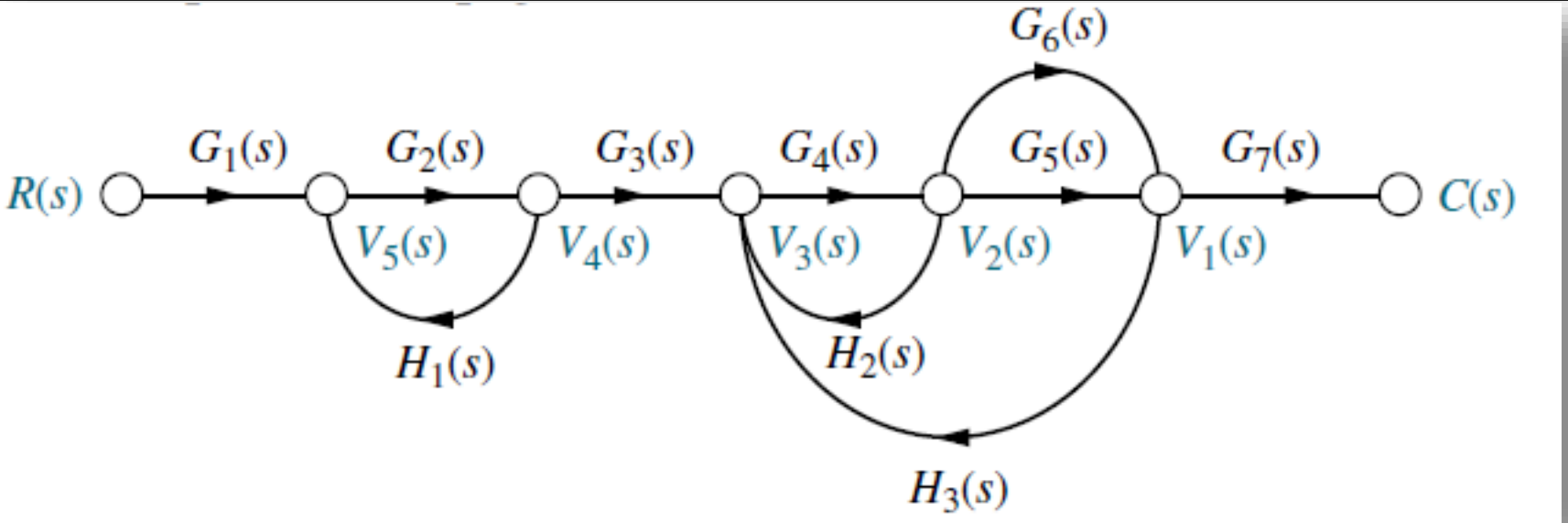
Signal Flow Graph (SFG) :



Signal Flow Graph (SFG) :

Definitions:



1.Forward Path: It is route from i/p to o/p node.

→ $G_1G_2G_3G_4G_5G_7$

→ $G_1G_2G_3G_4G_6G_7$

2. Forward Path Gain: The product of gains found by traversing a path from the input node to the output node of the signal flow graph in the direction of signal flow graph.

→ $G_1G_2G_3G_4G_5G_7$

→ $G_1G_2G_3G_4G_6G_7$

3. Non touching loops: Loops that do not have any nodes in common. In figure, G_2H_1 does not touch loops G_4H_2 , $G_4G_5H_3$ and $G_4G_6H_3$

Definitions:

4. Non touching loop gain: The product of loop gains from non touching loops taken two, three, four or more at a time. The non touching loop gain as:

→ G_2H_1 & G_4H_2

→ G_2H_1 & $G_4G_5H_3$

→ G_2H_1 & $G_4G_6H_3$

5. Loop Gain: The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once. The loop gains as:

→ G_2H_1

→ $G_4\ H_2$

→ $G_4G_5H_3$

→ $G_4G_6H_3$

Mason Gain Formula:

A signal flow graph (SFG) is a new method for finding a transfer function apart from block diagram reduction systems. This formula was invented by S J Mason in 1953. Hence, it is well known as "Mason's Gain Formula". It is given as:

$$Transfer\ Function = \frac{C(s)}{R(s)} = \frac{1}{\Delta} \boldsymbol{\Sigma}\, P_i \Delta_i$$

Where,

i = Number of Forward Paths

P_i = Gain of i^{th} Forward Path

Δ = 1 – (sum of all individual loop gains including self loop gains) + (sum of all gain products of two non touching loops) – (sum of all gain products of three non touching loops) + …

Δ_i = The value of Δ for the part of graph non touching to the i^{th} Forward Path

Signal Flow Graph (SFG) : conti...

Mason Gain Formula Steps:

1. Find out all possible forward paths i.e. P_1, P_2, P_3 … and their gains.
2. Find out all single loops i.e. P_{11}, P_{21}, P_{31} … P_{m1} [P_{11}= First single loop, P_{21}= Second single loop, P_{31} = Third single loop, ….]
3. Find out two non touching loops out of step 2 , P_{12}, P_{22}, P_{32} … P_{m2} with their gains if any [P_{12} is first product of two non touching loop, P_{22} is second product of two non touching loop, etc…]
4. Find out three non touching loops out of step 3, P_{13}, P_{23}, … P_{m3} with their gains if any [P_{13} is first product of thee non touching loop, P_{23} is second product of three non touching loop, etc…]
5. The value of $\Delta = 1 - (P_{11} + P_{21} + P_{31} + \ldots + P_{m1}) + (P_{12} + P_{22} + P_{32} + \ldots + P_{m2}) - (P_{13} + P_{23} + \ldots + P_{m3}) + \ldots$
6. Find the value of Δ_i (where i is the number of forward paths)
7. Find the transfer function.

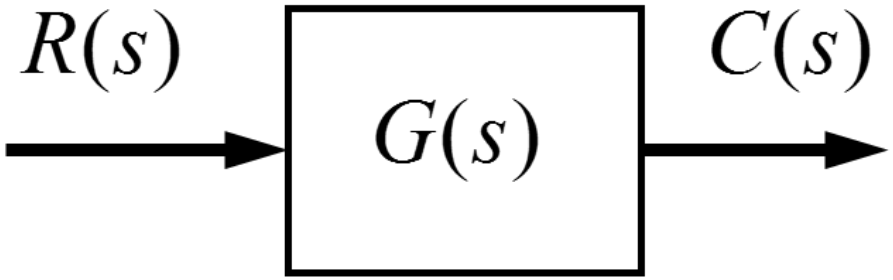
Signal Flow Graph (SFG) : conti...

Characteristics / Properties of SFG:

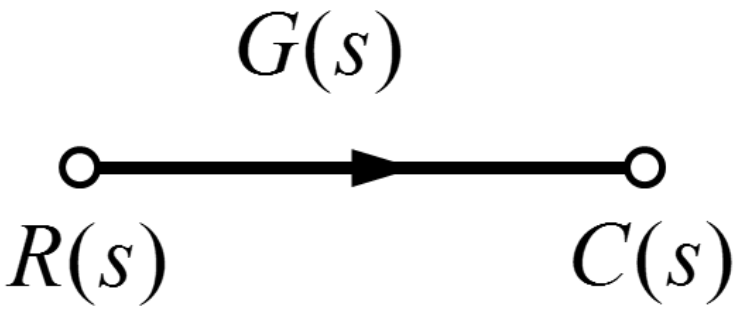
1. Signal flow applies to only linear systems.
2. A branch indicates the functional dependence of one signal on another. A signal passes through only in the direction specified by the arrow of the branch.
3. A node adds the signals of all incoming branches and transmits this sum to all outgoing branches.
4. A mixed node, which has both incoming and outgoing branches, may be treated as an output node (sink) by adding an outgoing branch of unity transmittance. Note, however, that we cannot change a mixed node to a source by this method.
5. The branch directing from node G_k to G_j represents dependence of the variable G_j on G_k but not the reverse.
6. The signal traveling along the branch G_k and G_j is multiplied by branch gain a_{kj} and signal $a_{kj}\ G_k$ is delivered at node G_j .
7. For a given system, a signal flow graph is not unique. Many different signal flow graphs can be drawn for a given system by writing the system equations differently.

BRTS & SFG Conversion:

block diagram:

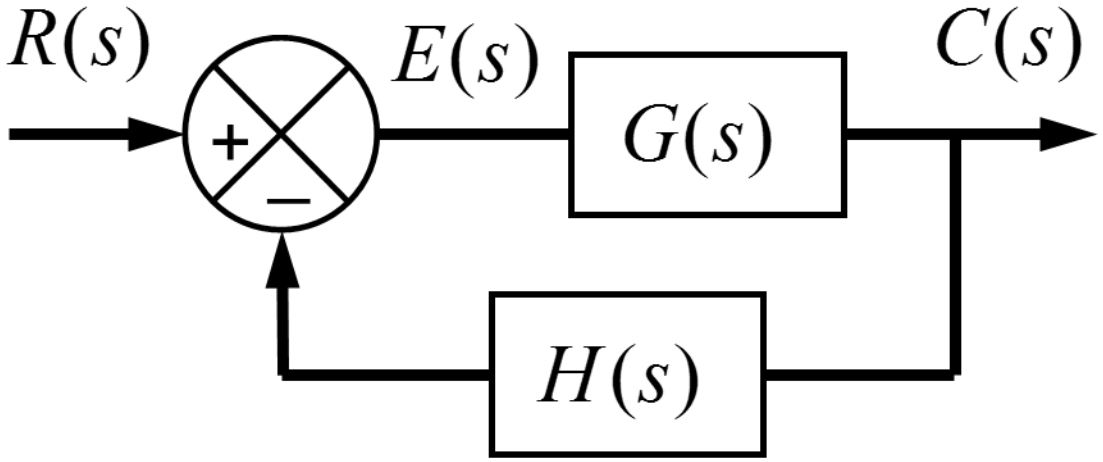


signal flow graph:

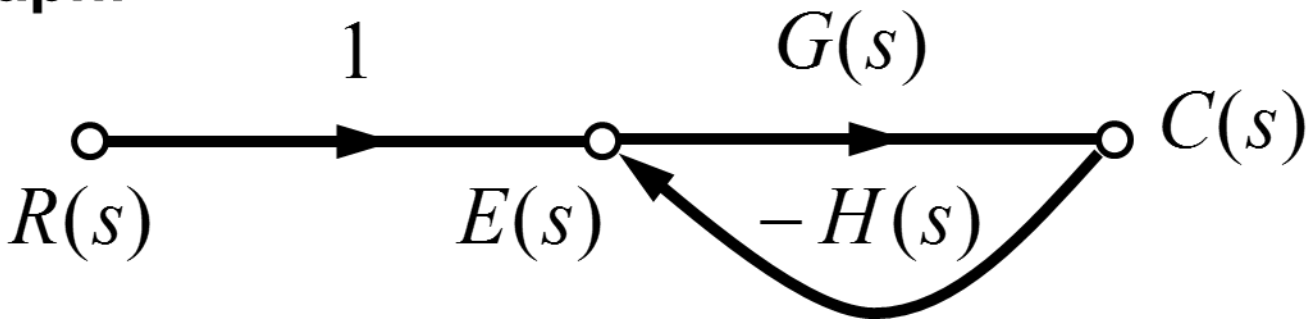


BRTS & SFG Conversion:

block diagram:



signal flow graph:



BRTS & SFG Conversion:

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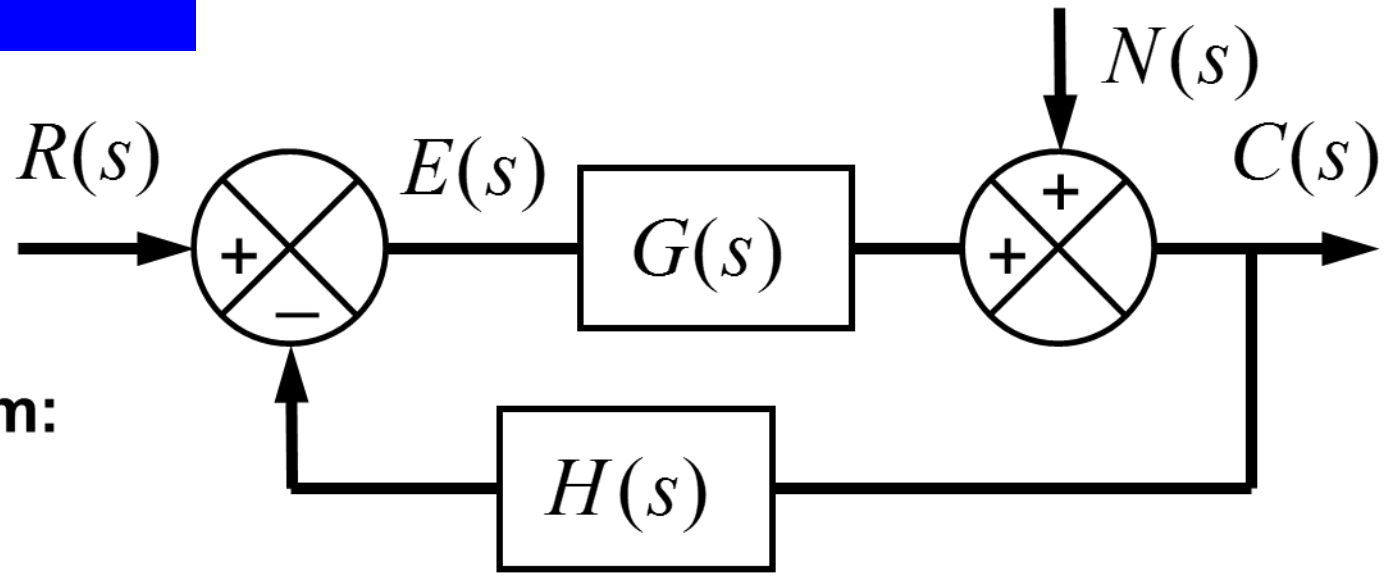
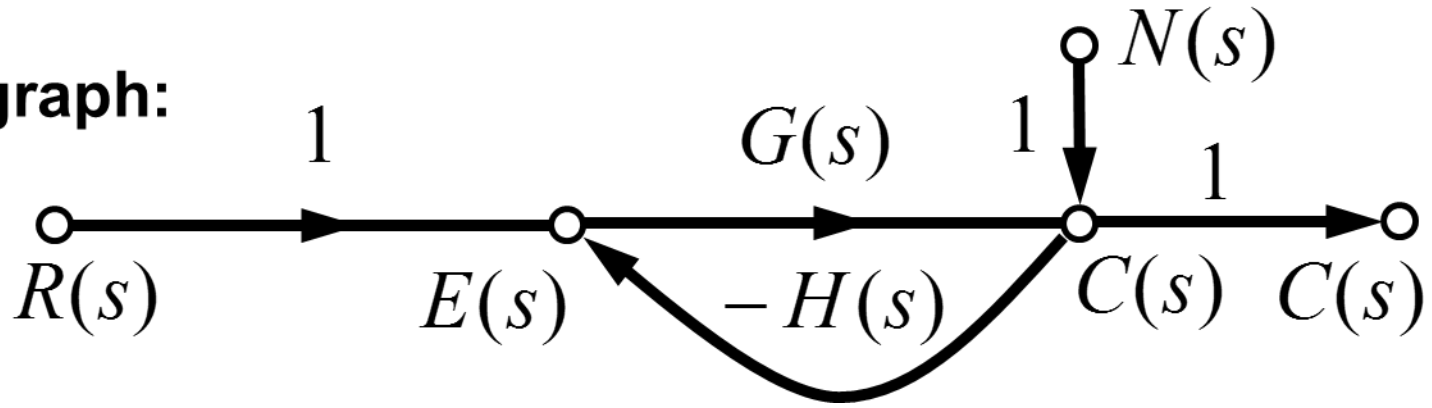


signal flow graph:



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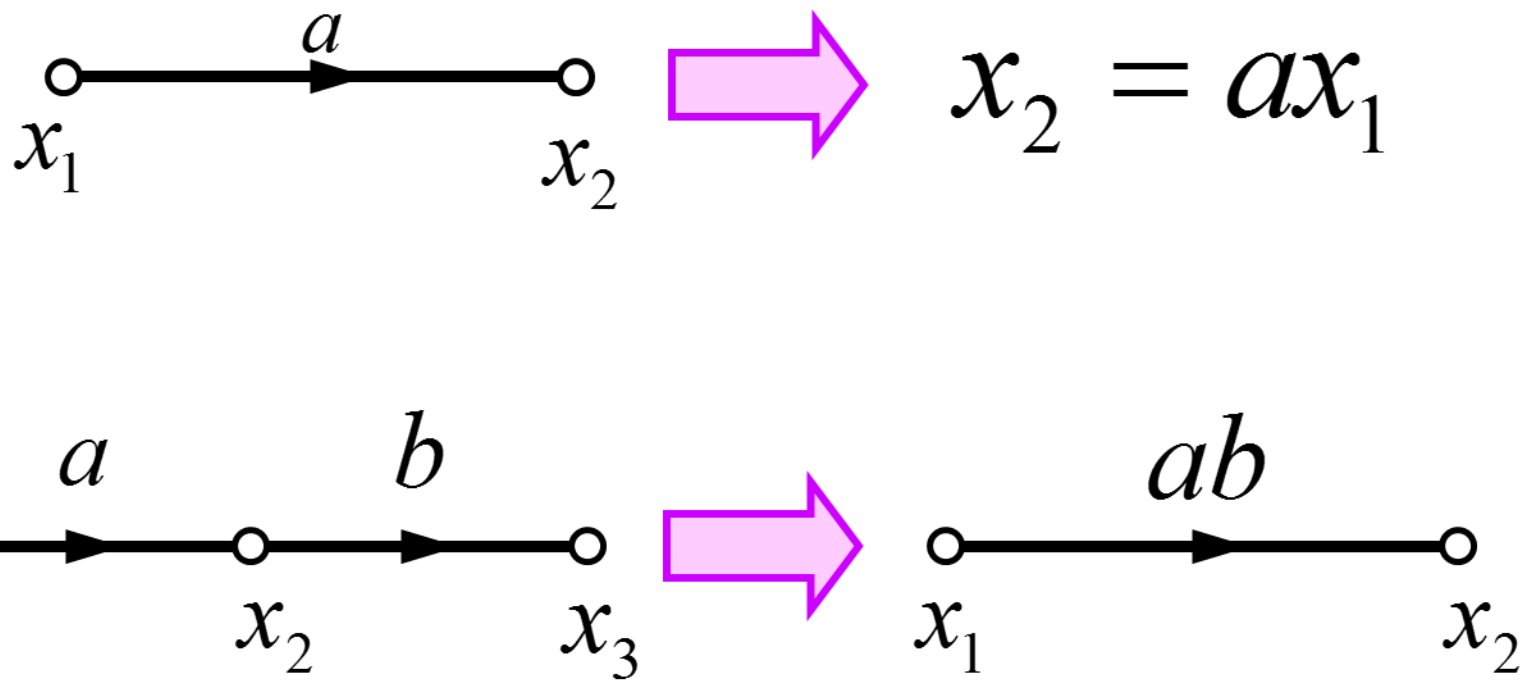

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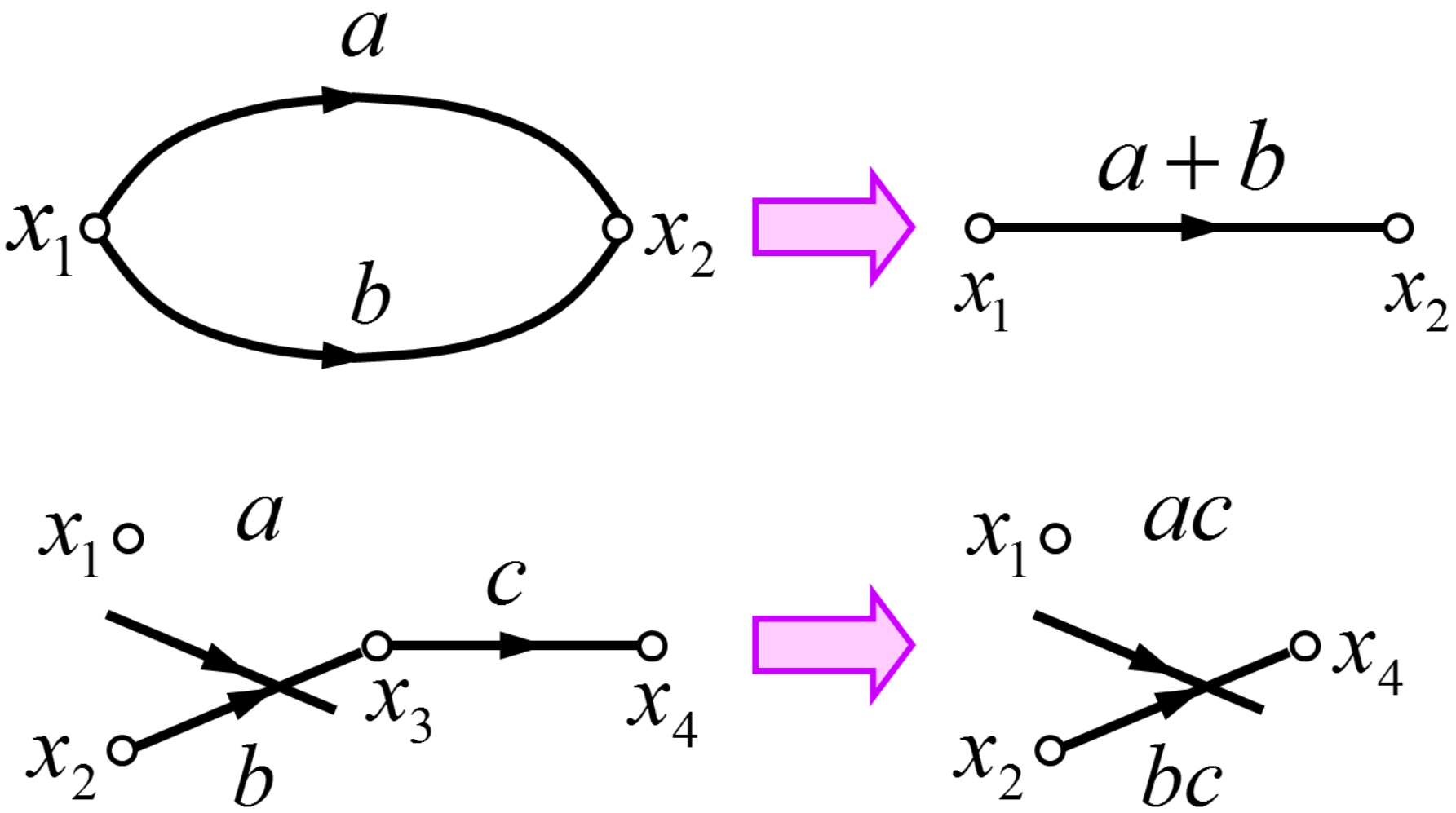

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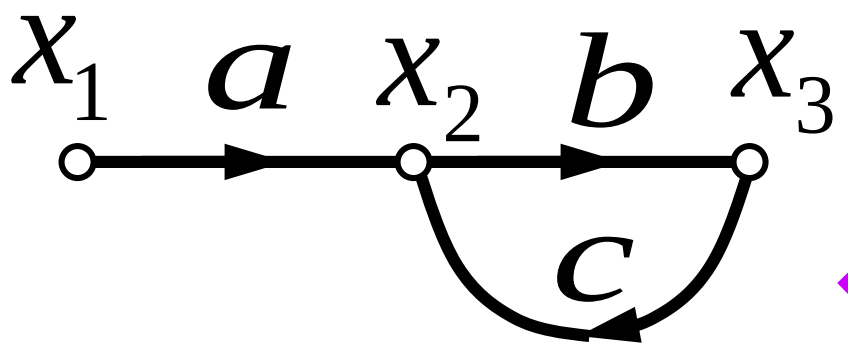
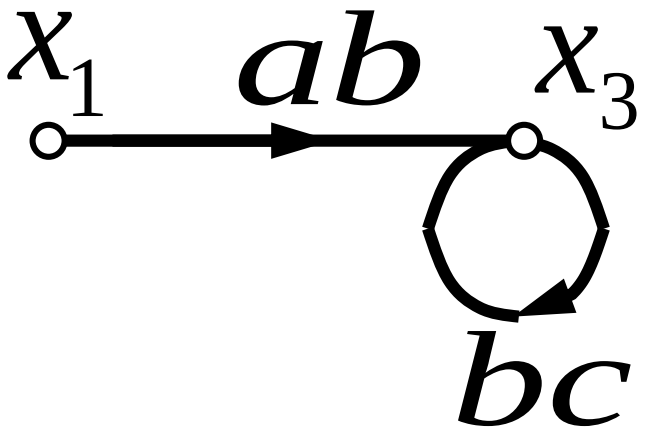
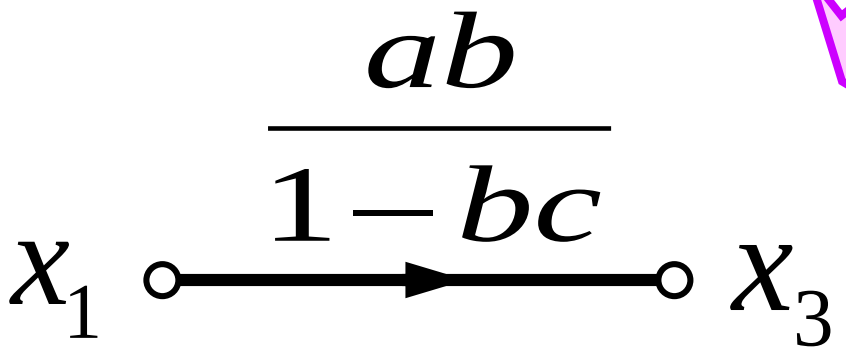


$$x_3 = bx_2$$

$$x_2 = ax_1 + cx_3$$



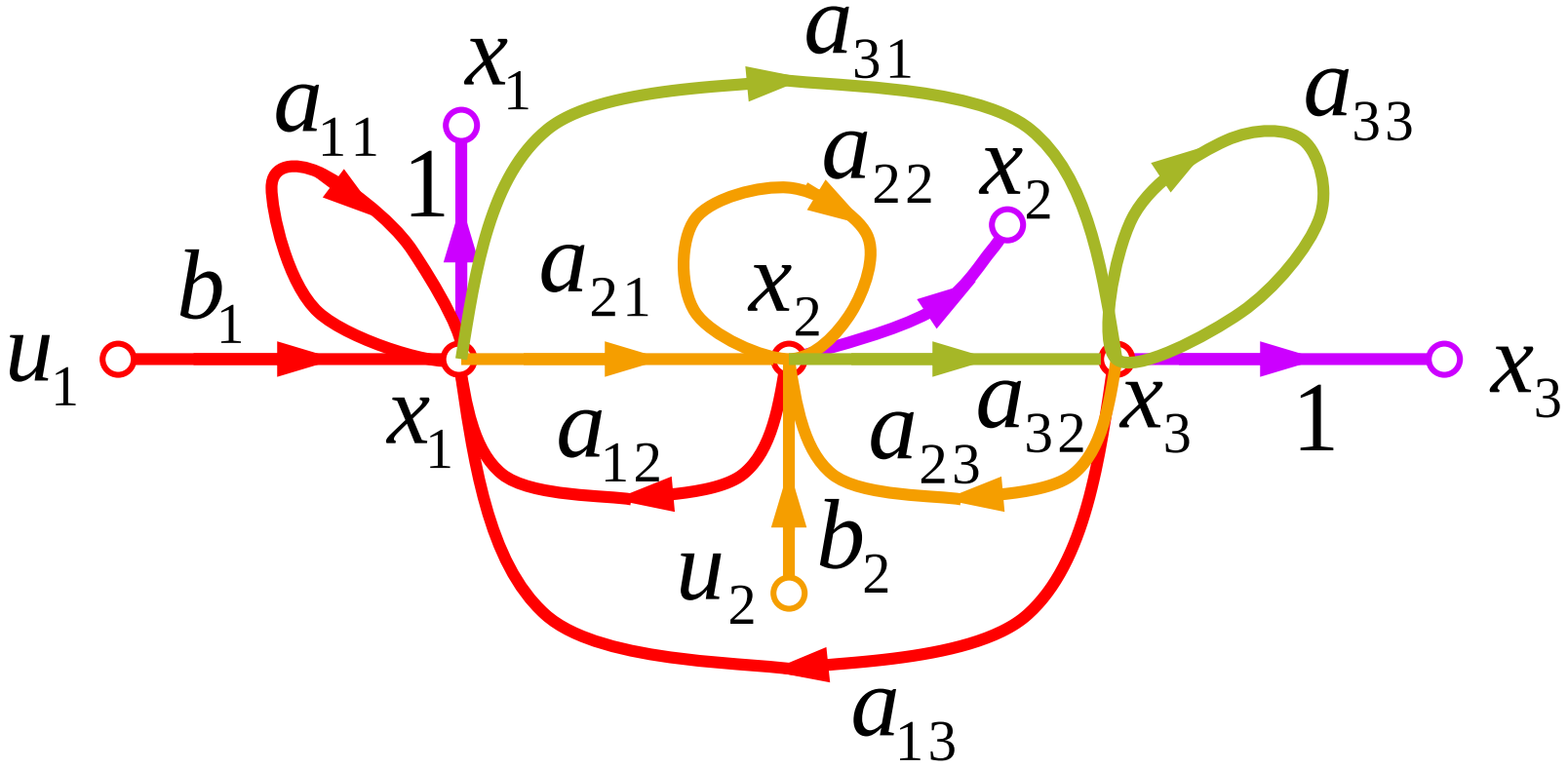
$$x_3 = abx_1 + bcx_3$$



$$\frac{x_3}{x_1} = \frac{ab}{1-bc}$$

BRTS & SFG Conversion:

$$x_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1u_1$$
$$x_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2u_2$$
$$x_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3$$



Thank You !