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Modeling and Analysis of Quadrotor using Sliding Mode Control

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Abstract— **In this paper, we present a model of a four rotor unmanned air vehicle (UAV) known as quadrotor aircraft. Quadrotors have generated considerable interest in both the control community due to their complex dynamics and military because of their advantages over regular air vehicles. The model is used to design a stable and accurate controller. Main aim of this paper is to achieve stable hover, with addition of variations in altitude based on human input for thrust. This paper explains the developments of a PD control method and sliding mode control for a fully-actuated subsystem of a quadrotor to obtain stability in flying the Quadrotor to a stable hovering position. The model has four input forces which are basically the thrust provided by each propeller.**

I. Introduction

The list of potential The unmanned aerial vehicles (UAVs) applications is endless and would include numerous surveillance, search and rescue applications as well as specific tasks such as fire fighting. Generally, UAV application is envisaged within the 3Ds environment which refers to dangerous, dull or dirty environments; in other words for occasions where it is not desirable to use a human pilot. There are also many potential indoor applications such as internal factory inspection, reconnaissance within an urban environment and observation of a structurally unsafe building. UAVs are expected to become a major part of the aviation industry over the next years, primarily enabled by developments in computer science, automatic control, robotics, communications and sensor technologies.

In this paper, we are studying the behaviour of the quadrotor. A quadrotor is a four propellers helicopter. This flying robot presents the main advantage of having quite simple dynamic features. Indeed, the quadrotor is a small vehicle with four propellers placed around a main body. The basic motions of a quadrotor are generated by varying the rotor speeds of all four rotors, thereby changing the lift forces. The helicopter tilts towards the direction of low lift rotor, which enables acceleration along that direction. Spinning directions of the rotors are set to balance the moments, therefore eliminating the need for a tail rotor.

The rotational speeds of the four rotors are independent. Thanks To this independence, its possible to control the pitch, roll and yaw attitude of the vehicle. Then, its displacement is produced by the total thrust of the four rotors whose direction varies according to the attitude.

One of the advantages of Quadrotors is the payload augmentation. They have more lift thrusts than conventional helicopters therefore they offer better payload. Moreover, they are potentially simpler to build and highly manoeuvrable. These advantages qualify a quadrotor as a good platform for autonomous unmanned aerial vehicle research ([1]-[8]).

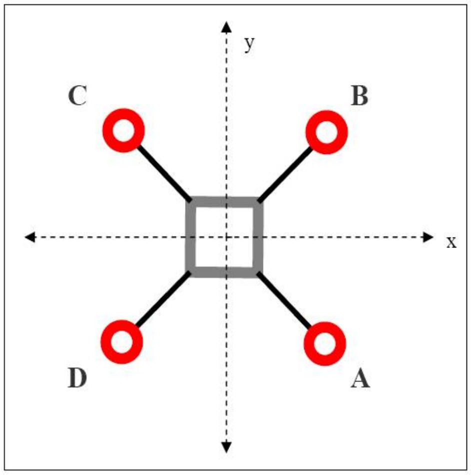

Fig. 1. Schematic view of Quadrotor

Quadrotor aerial robot can generate 6-DOF movement in the inertia frame through changing the motors rotational speed. Including translational motion along three coordinate axis (surge, sway and heave) and rotational motion around three axis (roll, pitch and yaw). Vertical motion of z-axis is achieved by increasing (or decreasing) speed of four motors altogether with the same quantity. When the total of thrust equal to the self weight, the quadrotor aerial robot change to a hoverable robot. The change of pitch angle is achieved by a difference thrust between the front and the rear rotors and simultaneously to maintain the total thrust while the change of roll angle result from differences between the left and right rotor by the same way, respectively. Yaw rotation can be achieved by the difference in the counter-torque between each pair (A,C and B,D) of rotors. And maintaining the total thrust unchanged to

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avoid the up-down motion.

A quadrotor is an under-actuated system with four independent inputs and six coordinate outputs. Several methods have been proposed to control a quadrotor. Feedback linearization method was first used by Mistler and et al. to make the quadrotor track a reference trajectory [1]. Castillo and et al. applied nested saturations control to move a quadrotor to a position and stabilize its attitude [2], [3]. Altug and et al. used the backstepping method to stabilize a quadrotor by keeping the positions and the yaw angle constant and the pitch and the roll angle zero [4], [5].

II. Dynamic Model

A quadrotor is an under actuated aircraft with fixed pitch angle four rotors as shown in Fig. 2

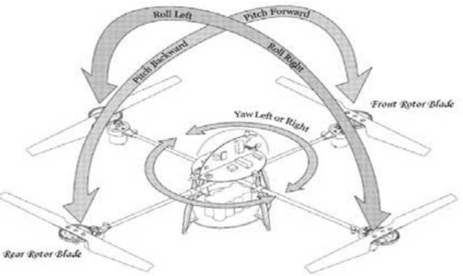

Fig. 2. The Quadrotor Rotorcraft

The four-rotor rotorcraft does not have a swashplate like twin rotor helicopter. It does not need any blade pitch control. The collective input is the sum of the thrusts of each motor. Pitch movement is obtained by increasing (reducing) the speed of the rear motor while reducing (increasing) the speed of the front motor. The roll movement is obtained similarly using the lateral motors. The yaw movement is obtained by increasing (decreasing) the speed of the front and rear motors while decreasing (increasing) the speed of the lateral motors. This should be done while keeping the total thrust constant.

For example, the craft will move in the positive x-direction by reducing the thrust from motor A (Figure 1) (by reducing the power) and simultaneously increasing the thrust (by increasing the power) from motor C. The thrust from motor B and D must be increased so that the craft maintains constant altitude while moving along the desired path. More complex movements can be achieved by varying the power delivered to all four motors.

The generalized coordinates for the rotorcraft are

$$q = (x, y, z, \psi, \theta, \varphi) \in R6$$

where (x, y, z) denote the position of the centre of mass of the four-rotor rotorcraft relative to the frame I, and (ψ, θ, φ) are the three Euler angles (yaw, pitch and roll angles) and represent the orientation of the rotorcraft.

The dynamic model of the quadrotor helicopter can be obtained via a Lagrange approach and a simplified model is given as follow [4]

$$\ddot{x} = \frac{\sum_{i=1}^{4} F_i(cos\phi sin\theta cos\varphi - cos\theta sin\varphi) - k_1\dot{x}}{m}$$

$$\ddot{y} = \frac{\sum_{i=1}^{4} F_i(sin\phi sin\theta cos\varphi - cos\phi sin\varphi) - k_2\dot{y}}{m}$$

$$\ddot{z} = \frac{\sum_{i=1}^{4} F_i(cos\phi cos\varphi) - mg - k_3\dot{z}}{m}$$

$$\ddot{\theta} = \frac{L(-F_1 - F_2 - F_3 - F_4 - K_4\dot{\theta})}{J_1}$$

$$\ddot{\varphi} = \frac{L(-F_1 + F_2 + F_3 - F_4 - K_5\dot{\varphi})}{J_2}$$

$$\ddot{\phi} = \frac{L(-F_1 - F_2 + F_3 - F_4 - K_6\dot{\phi})}{J_1}$$

The forces on the motors are given by the F_i terms. The moments of inertia of the craft with respect to the axes are given by the J_i terms (where x corresponds to 1, y corresponds to 2, and z corresponds to 3). The K_i terms represent the drag coefficients, which can be ignored for simplicity. C is a force to moment scaling factor. The centre of gravity is assumed to be on the origin.

This quadrotor helicopter model has six outputs $(x, y, z, \theta, \phi, \varphi)$ while it only has four independent inputs, therefore the quadrotor is an under-actuated system. We are not able to control all of the states at the same time. A possible combination of controlled outputs can be x, y, z and φ in order to track the desired positions, move to an arbitrary heading and stabilize the other two angles, which introduces stable zero dynamics into the system [4], [5].

III. Altitude And Yaw Control

In this section, we present our control strategies to stabilize the quad rotor on its hovering position. We will apply PD control strategy to attain desired altitude and yaw angle. The control of the vertical position can be obtained by using the following control input[6].

$$u_1 = \frac{r_1 + mg}{cos\theta cos\varphi}$$

where,

$$r_1 = k_{zd}\dot{z} - k_{zp}(z - z_d)$$

where k_{zd}, k_{pd} are positive constants and z_d is the desired altitude. The yaw angular position can be controlled by applying

$$u_4 = k_{\varphi d}\dot{\varphi} + k_{\varphi p}(\varphi - \varphi_d)$$

where $k_{\psi d}$ and $k_{\psi p}$ are the deferential and proportional gains of the PD controller and ψ_d is the desired yaw angle. Now, by choosing proper value of

$k_{zd}, k_{pd}, k_{\psi d}, and k_{\psi p}$, we can ensure the well damped response in vertical direction and yaw axis. We can attain desirable hovering altitude of the quadrotor rotorcraft.

IV. Sliding Mode Controller Design

The quadrotor model can be divided into two subsystems: a fully-actuated subsystem and an under actuated subsystem.

A fully-actuated subsystem

$$\begin{bmatrix} \ddot{z} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} u_1 cos\theta cos\varphi - g \\ u_4 \end{bmatrix} + \begin{bmatrix} \frac{-k_3 z}{m} \\ \frac{-k_6 \dot{\phi}}{I_3} \end{bmatrix}$$

and an under-actuated subsystem

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} u_1 cos\phi & u_1 sin\phi \\ u_1 sin\phi & -u_1 cos\phi \end{bmatrix} \begin{bmatrix} sin\theta cos\psi \\ sin\psi \end{bmatrix} + \begin{bmatrix} \frac{-k_1 \dot{x}}{m} \\ \frac{-k_2 \dot{y}}{m} \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} u_2 \\ u_1 \end{bmatrix} + \begin{bmatrix} \frac{-lk_4 \dot{\theta}}{I_1} \\ \frac{-lk_5 \dot{\psi}}{I_2} \end{bmatrix}$$

Controller for the fully actuated subsystem:

For the fully actuated subsystem we can easily construct a rate bounded PID controller and a sliding mode controller to move states z and φ to their desired values z_d and φ_d respectively. The desired control input for z is given by

$$u_{1d} = \frac{k_{zp}(z_d - z) - k_{zd}\dot{z} + mg}{cos\theta cos\varphi}$$

A rate bounded control u_1 will converge to u_{1d} [7],

$$\dot{u}_1 = ksat\left(\frac{k_1(u_{1d} - u_1)}{\epsilon}\right)$$

where $sat(\cdot)$ is the saturation function defined as

$$sat(x) := \begin{cases} 1, & x > 1 \\ x & -1 \leq x \leq 1 \\ -1 & x < -1 \end{cases}$$

For Φ , a sliding mode control is designed to make Φ converge to its desired value Φ_d quickly

$$u_4 = c_\Phi \dot{\Phi} - M_\Phi sgn(s_\Phi) - k_\Phi s_\Phi$$

where k_{zp} k_{zd} c_Φ M_Φ *and* K_Φ are controller parameters to be determined and all positive, and $s_\Phi = c_\Phi(\Phi - \Phi_d) + \dot{\Phi}$ the designed stable sliding surface for Φ. $sgn(\cdot)$ is a continuous approximation of the sign function and defined as

$$sgn(x) = (\tfrac{2}{\pi}) arctan(\mu x)$$

where $\mu > 0$ is a positive constant and the approximation error can be decreased by increasing μ. This approximation is used to avoid the chattering in the sliding mode control. It is trivial to show that the control laws [3] and [4] can make z and Φ asymptotically converge to their desired values z_d and Φ respectively.

The future works involves control using of the sliding mode controller for under actuated subsystem of quadrotor. Designing of sliding mode observer for quadrotor and observer based output tracking controllers.

V. Simulation Results

As a part of simulation tool MATLAB is used and we taking the thrust forces of all the four motors as four control input to quadrotor. The craft can increase in altitude by simultaneously increasing the thrust from all motors. Likewise, the craft can descend (if already airborne) by simultaneously decreasing the thrust from all motors. Then we applied PD control to attain the desired altitude and keep the quadrotor in hovering position.

A. Simulation 1

When all motors are left at the same thrust (say 1 Newton), then the quadrotor will rise if the total thrust produced is larger than the weight of quadrotor.

B. Simulation 2

We simulate movement of the craft when the thrust from motor A is increased by 10% and the thrust from motor C is decreased by 10%. The thrust from motors B and D remains constant.

C. Simulation 3

Quadrotor altitude and yaw control using sliding mode for fully actuated subsystem has been simulated using soft computing techniques.

VI. Conclusion and Future Work

This paper presented two control strategies to bring the quadrotor to a stable hovering position. In both the technique quadrotor was brought to desirable altitude with required yaw angle. While using sliding

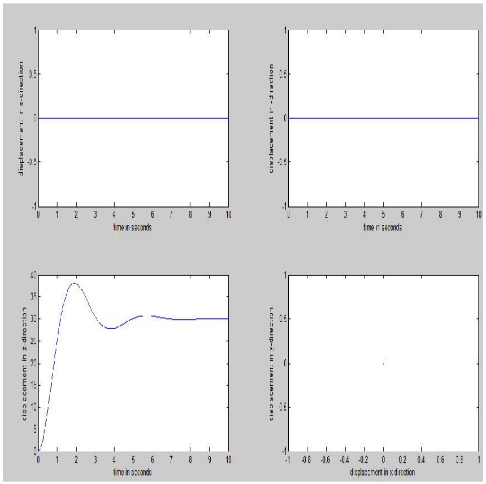

Fig. 3. Result of Simulation 1

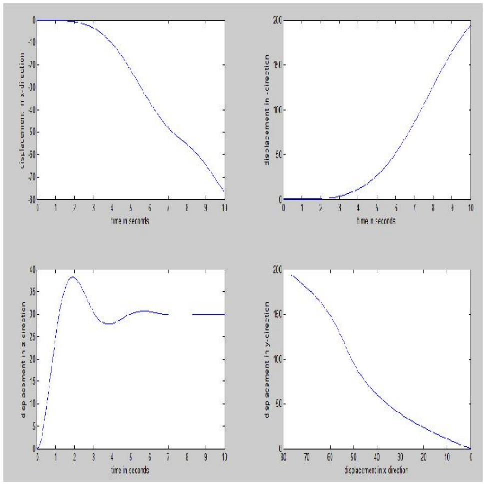

Fig. 4. Result of Simulation 2

mode approach we divided our system into a fully-actuated subsystem and an under-actuated subsystem. In sliding mode control strategy the resulting system and controller mathematical models were converted to their respective Simulink models for ease of simulations. While for PD controller we have used MATLAB to solve the system of six differential equations that simulate the Quadrotors motion.

Also one can go for discontinuous feedback control or Model Predictive Control for controlling of the quadrotor.

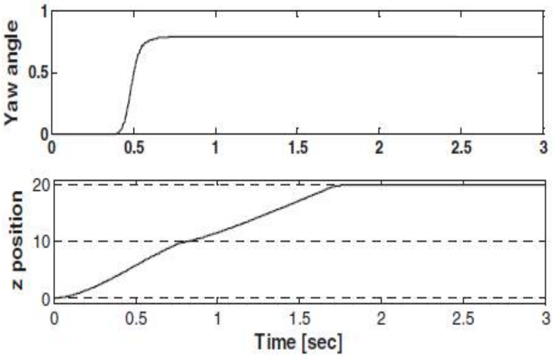

Fig. 5. Result of Simulation 3

VII. Acknowledgements

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