University of Jeddah
Faculty of Engineering
Department of Electrical & Computer Engineering

Summer Semester – 2016

Electromagnetic Fields (ECE 308)

Lecture 5 – Electrostatics - II

Ankit Patel
majorankit@gmail.com
www.majorankit.wix.com/majorankit

Charge & Current Distributions:

Surface charge density:

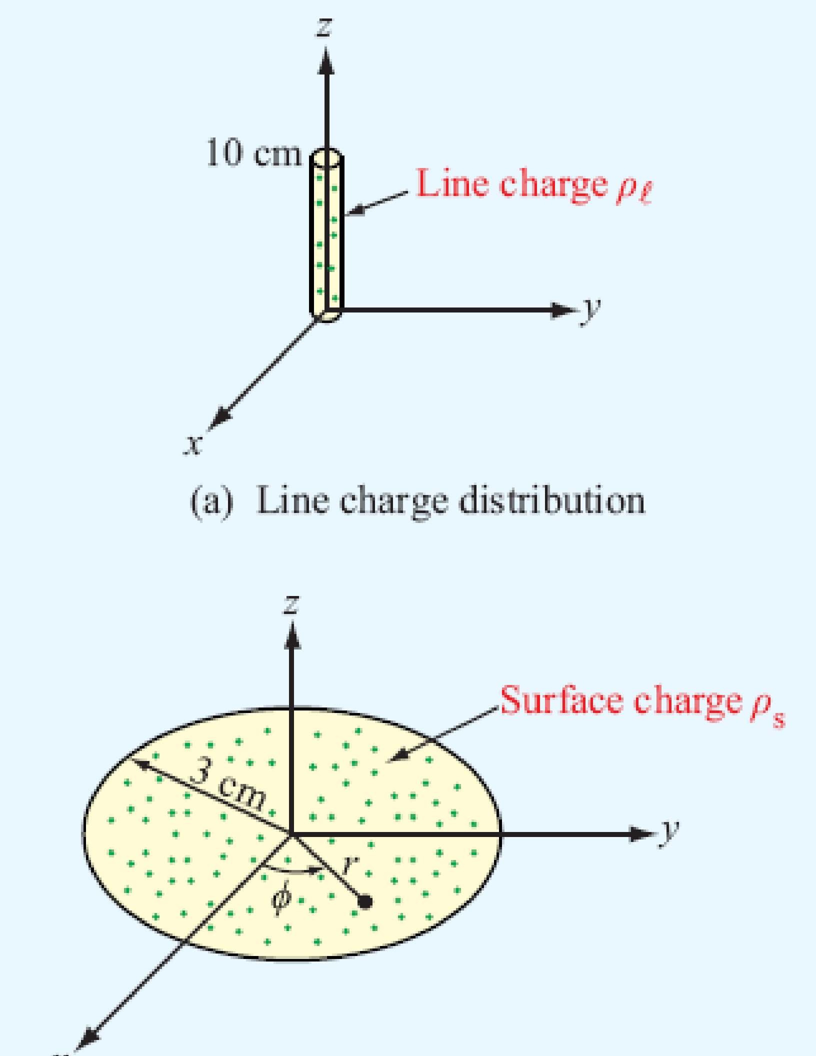
$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v} = \frac{dq}{dv}$$

$$Q = \int_{v} \rho_v \, dv$$

In some cases, particularly when dealing with conductors, electric charge may be distributed across the surface of a material, in which case the quantity of interest is the surface charge density, ρ_s, defined as

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta q}{\Delta s} = \frac{dq}{ds}$$

where Δq is the charge present across an elemental surface area Δs.



(a) Line charge distribution

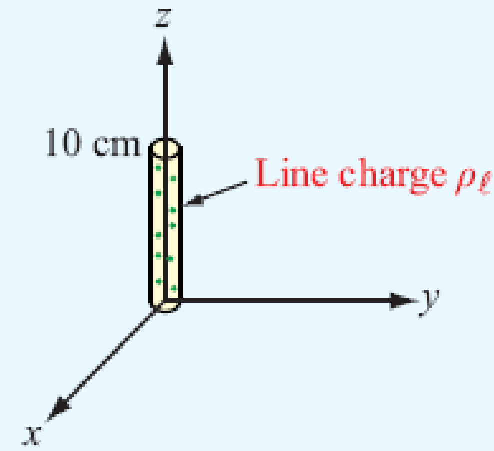
(b) Surface charge distribution

Charge & Current Distributions:

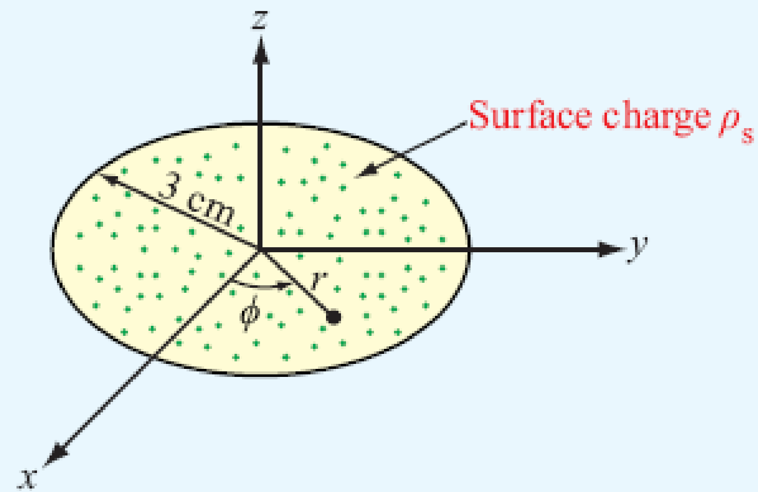
Line charge density:

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v} = \frac{dq}{dv}$$

$$Q = \int_v \rho_v \, dv$$



(a) Line charge distribution



(b) Surface charge distribution

If the charge is, for all practical purposes, confined to a line, which need not be straight, we characterize its distribution in terms of the line charge density, ρ_l, defined as:

$$\rho_\ell = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl}$$

Charge & Current Distributions:

Current density:

Consider a tube with volume charge density $\boldsymbol{\rho_v}$. The charges in the tube move with velocity **u** along the tube axis. Over a period of **Δt**, the charges move a distance **Δl = u Δt**. The amount of charge that crosses the tube's cross-sectional surface **Δs'** in time **Δt** is therefore,

$$\boxed{\Delta q' = \rho_v \, \Delta \mathcal{V} = \rho_v \, \Delta l \, \Delta s' = \rho_v u \, \Delta s' \, \Delta t.}$$

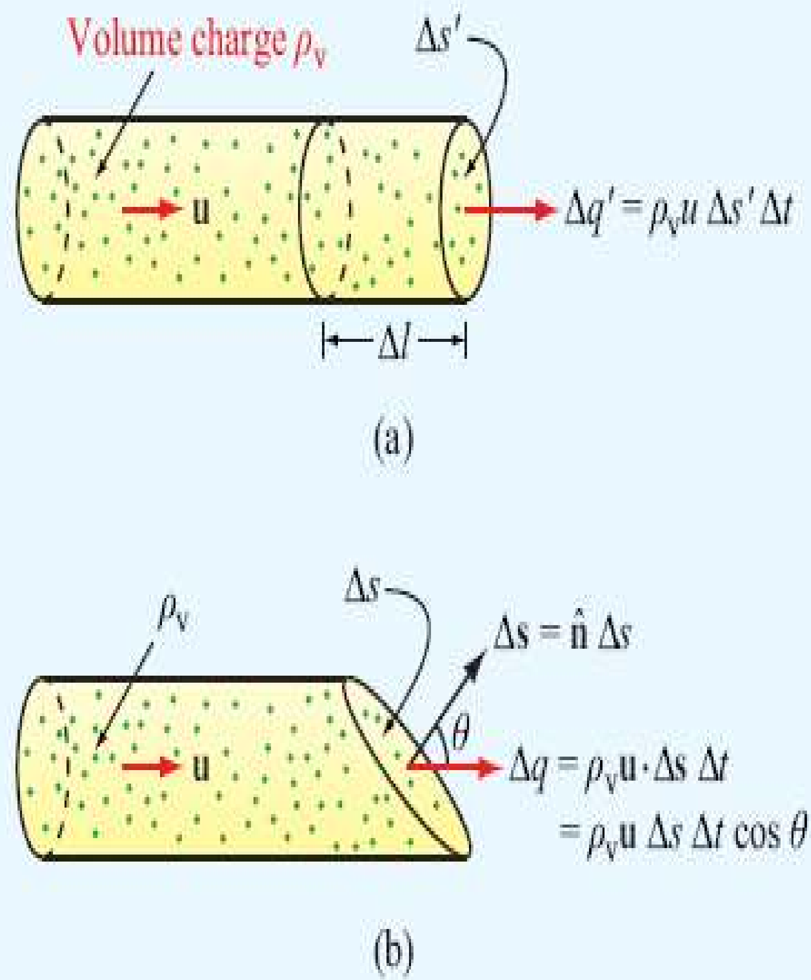


Figure 4-2 Charges with velocity **u** moving through a cross section $\Delta s'$ in (a) and Δs in (b).

Charge & Current Distributions:

Current density:

Now consider the more general case where the charges are flowing through a surface **Δs** with normal to **unit vector n, not necessarily parallel to u**. In this case, the amount of charge **Δq** flowing through **Δs**, is given by:

$$\boxed{\Delta q = \rho_v \mathbf{u} \cdot \Delta \mathbf{s}\, \Delta t,}$$

The corresponding total current flowing in the tube is

$$\boxed{\Delta I = \frac{\Delta q}{\Delta t} = \rho_v \mathbf{u} \cdot \Delta \mathbf{s} = \mathbf{J} \cdot \Delta \mathbf{s},}$$

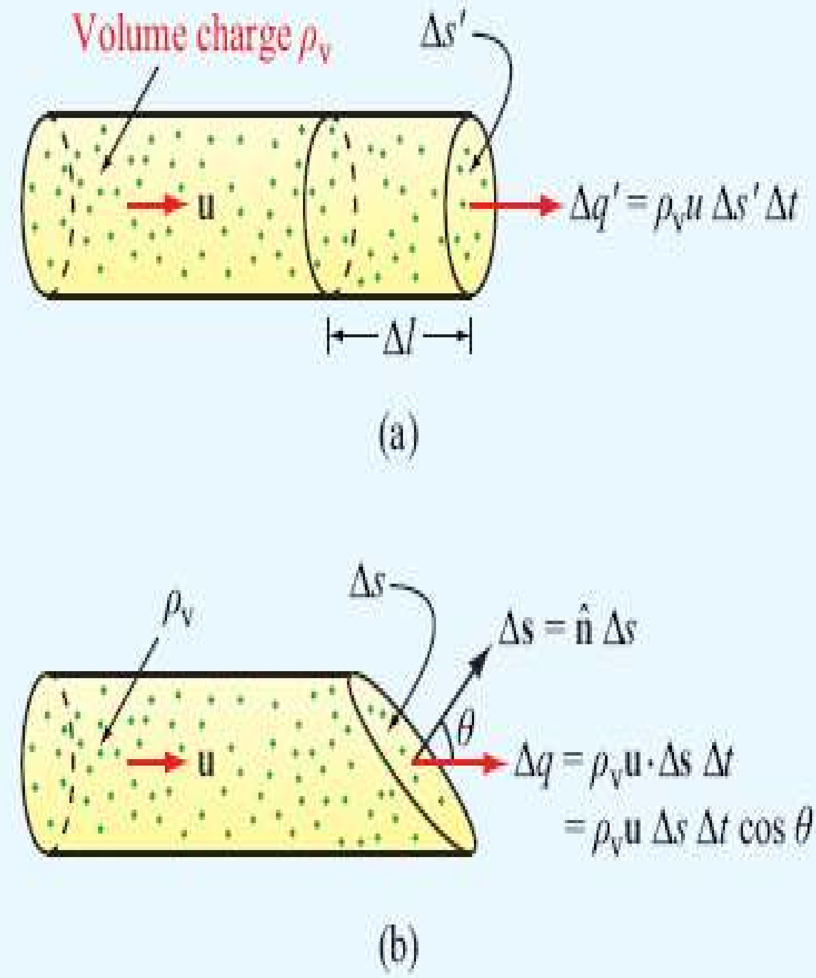


Figure 4-2 Charges with velocity **u** moving through a cross section $\Delta s'$ in (a) and Δs in (b).

Charge & Current Distributions:

Current density:

$$\Delta I = \frac{\Delta q}{\Delta t} = \rho_v \mathbf{u} \cdot \Delta \mathbf{s} = \mathbf{J} \cdot \Delta \mathbf{s},$$

Where, $\mathbf{J} = \rho_v \mathbf{u}$ is defined as the **Current Density** in ampere per square meter. Generalized to the arbitrary surface S, the total current flowing through it is

$$I = \int_S \mathbf{J} \cdot d\mathbf{s}$$

Convection current density:

"When a current is due to the actual movement of electrically charged matter, it is called a Convection Current, and J is called a Convection Current Density."

For Example: A wind driven charged cloud, gives rise to a convection current. In some cases, the charged matter constituting the convection current consists solely of charged particles, such as the electron beam of a scanning electron microscope or ion beam in plasma propulsion system.

Charge & Current Distributions:

Current density:

Conduction current density:

When a current is due to the movement of charged particles relative to their host material, J is called a **Conduction Current Density**. In a metal wire, for example, there are equal amounts of positive charges (in atomic nuclei) and negative charges (in the electron shells of the atoms). None of the positive charges and few of the negative charges can move, only those electrons in the outermost electron shells of the atoms can be pushed from one atom to the next if a voltage is applied across the ends of the wire.

"This movement of electrons from atom to atom constitutes a Conduction Current. The electrons that emerge from the wire are not necessarily the same electrons that entered the wire at the other end."

Coulomb's Law:

Coulomb's law was first introduced for electrical charges in air and later generalized to material media, imply

[1] An isolated charge q induces an Electric field E at every point in space and at any specific point P, E is given By:

$$\boxed{\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\epsilon R^2}}$$

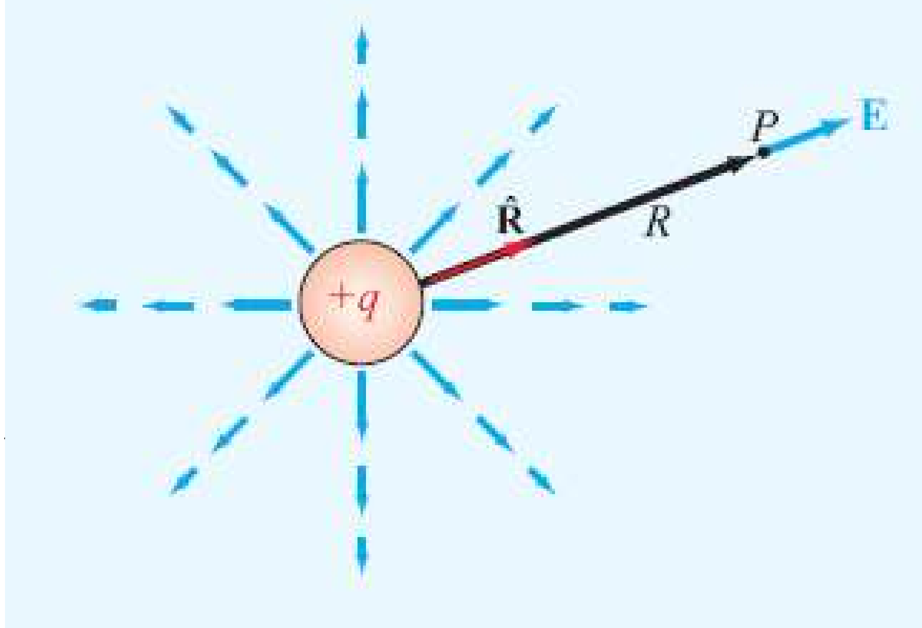


Figure 4-3 Electric-field lines due to a charge q.

where, $\hat{\mathbf{R}}$ is a unit vector pointing from q to P, and R is the distance between them. Ɛ is the electrical permittivity of the medium containing the observation point P.

Coulomb's Law:

[2] In the presence of an electric field E at a given point in space, which may be due to a single charge or a distribution of charges, the force acting on a test charge q' when placed at P is,

$$\boxed{\mathbf{F} = q'\mathbf{E}}$$

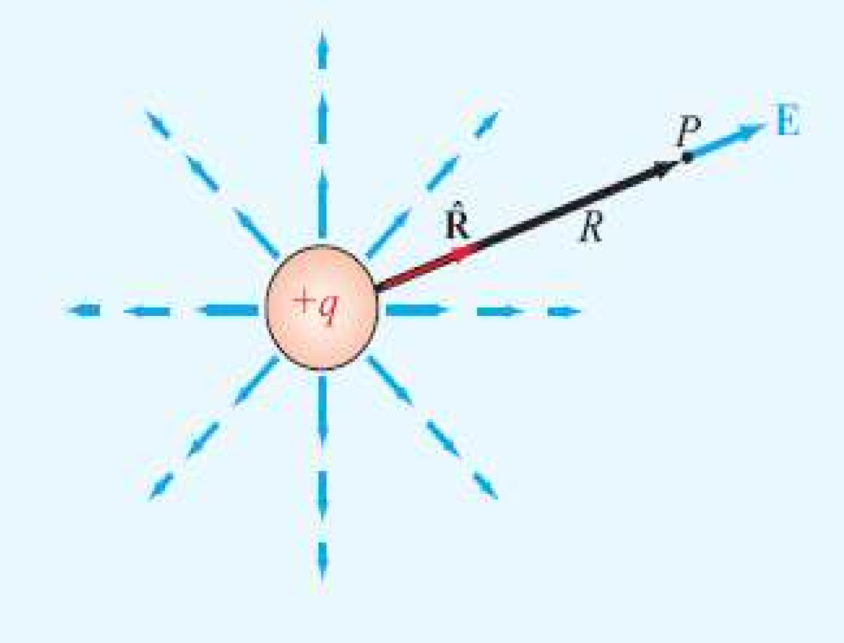


Figure 4-3 Electric-field lines due to a charge q.

The unit of F is Newton (N),

The unit of q' is Coulomb (C)

So, the unit of E is N/C.

Coulomb's Law:

For a material with electrical permittivity ε, the electrical field quantities D and E are related by:

$$D = \varepsilon E$$

Where,

$$\varepsilon = \varepsilon_r \varepsilon_0$$

Where, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the electrical permittivity of free space, and $\boldsymbol{\varepsilon_r}$ is called **relative permittivity** (or **dielectric constant**) of the material. For most materials and under a wide range of conditions, ε is independent of both the magnitude and direction of E.

→ If **ε is independent of the magnitude of E**, then the material is said to be **Linear**, because D and E are related linearly, and if it **independent of direction of E**, the material is said to be **Isotropic**.

Thank you !