University of Jeddah
Faculty of Engineering
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Electromagnetic Fields (ECE 308)

Lecture 7 – Electrostatics - IV

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Electric Scalar Potential:

The operation of an electric circuit usually is described in terms of the currents flowing through its branches and the voltages at its nodes. The voltage difference V between two points in a circuit represents the amount of work, or **Potential Energy**, required to move a unit charge from one to the other.

"The term voltage is short for *voltage potential* and synonymous with **Electrical Potential**".

Even though when analyzing a circuit we may not consider the electric fields present in the circuit, it is in fact the existence of these fields that gives rise to voltage differences across circuit elements such as resistors or capacitors. The relationship between the electric field **E** and the electric potential **V**, is to be defined in upcoming slides.

Electric Potential as a Function of Electric Field:

Consider a simple case having a positive charge q in a uniform Electric field $\mathbf{E} = -\hat{\mathbf{y}}E$ in the $-y$ direction. The presence of the field E exerts a force F_e = qE on the charge in the $-y$ direction. To move

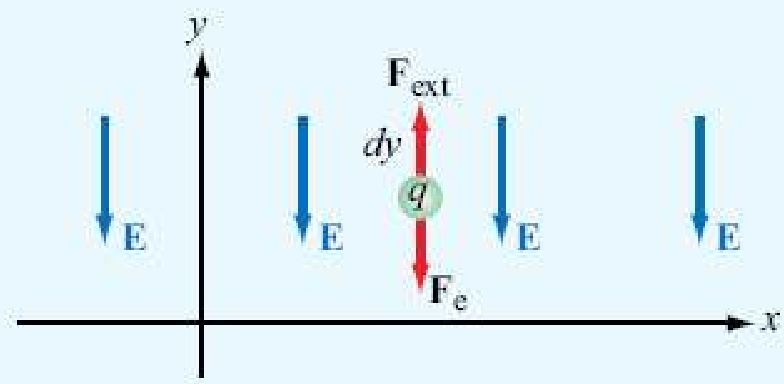

Figure 4-11 Work done in moving a charge q a distance dy against the electric field $\mathbf{E}$ is $dW = qE\,dy$.

the charge along the +y direction (against the force F_e), we need to provide an external force F_{ext} to counteract Fe, which requires the expenditure of energy. To move q without acceleration (at constant speed), the net force acting on the charge must be zero, which means that $F_{ext} + F_e = 0$, or

$$\mathbf{F}_{ext} = -\mathbf{F}_e = -q\mathbf{E}.$$

Electric Potential as a Function of Electric Field:

The work done, or energy expended, in moving any object a vector differential distance ***dl*** while exerting a force F_{ext} is

$$dW = \mathbf{F}_{ext} \cdot d\mathbf{l} = -q\mathbf{E} \cdot d\mathbf{l}$$

Work, or Energy is measured in Joule (J). If the charge is moved a distance *dy* along unit vector y, then

$$dW = -q(-\hat{\mathbf{y}}E) \cdot \hat{\mathbf{y}}\, dy = qE\, dy.$$

The differential electrical potential energy **dW** per unit charge is called the **differential electric potential** (or **differential voltage**) **dV**. That is

$$dV = \frac{dW}{q} = -\mathbf{E} \cdot d\mathbf{l} \qquad \text{(J/C or V)}.$$

Electric Potential as a Function of Electric Field:

The potential difference corresponding to moving a point charge from point P1 to point P2 is obtained by integrating along any path between them.

$$\int_{P_1}^{P_2} dV = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l},$$

$$V_{21} = V_2 - V_1 = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l},$$

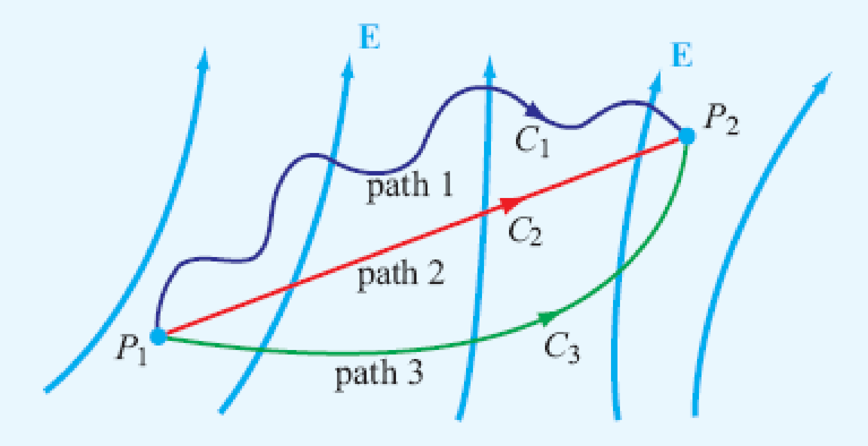

Figure 4-12 In electrostatics, the potential difference between P_2 and P_1 is the same irrespective of the path used for calculating the line integral of the electric field between them.

Where V1 and V2 are the electric potentials at points P1 and P2. The result of the line integral on the right hand side of above equation is independent of the specific integration path that connects points P1 and P2. This follows the laws of conservation of energy.

Electric Potential as a Function of Electric Field:

The voltage difference between two nodes in an electric circuit has the same value regardless of which path in the circuit we follow between the nodes. Moreover, **KVL states** that the net voltage drop around a closed loop is zero. If we go from P1 to P2 by path 1 in fig. and then return from P2 to P1 by path 2, the right hand side of eq. of potential difference becomes a closed contour and the left hand side vanishes. In fact, **the line integral of the electrostatic field E around any closed contour C is zero.**

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0 \qquad \text{(electrostatics)}.$$

"A vector field whose line integral along any closed path is zero is called a conservative or an Irrotational field. Hence, the electrostatic field E is conservative".

Electric Potential due to Point Charge:

The coulomb's law states that

$$\mathbf{E} = \hat{\mathbf{R}} \frac{q}{4\pi\epsilon R^2} \quad \text{(V/m)}$$

The field is radially directed and decays quadratically with the distance R from the observer to the charge.

As per the previous discussion, it is clear that the choice of integration path between the end points is arbitrary. Hence, we can conveniently choose the path to be along the radial direction in unit vector R,

$$V = -\int_{\infty}^{R} \left(\hat{\mathbf{R}} \frac{q}{4\pi\epsilon R^2}\right) \cdot \hat{\mathbf{R}}\, dR = \frac{q}{4\pi\epsilon R}$$

If the charge q is at a location other than the origin, say at position vector R1, then V at observation position vector R becomes

$$V = \frac{q}{4\pi\epsilon |\mathbf{R} - \mathbf{R}_1|}$$

Where |R – R1| is the distance between the observation point and the location of the charge q. The principle of superposition applied previously to the electric field **E** also applies to the electric potential V.

$$V = \frac{1}{4\pi\epsilon} \sum_{i=1}^{N} \frac{q_i}{|\mathbf{R} - \mathbf{R}_i|}$$

Electric Potential due to Continuous Distribution:

To obtain expressions for the electric potential V due to continuous charge distributions over a volume *v′*, across a surface *S′*, or along a line *l′*, we replace q_i in above equation gives,

$$V = \frac{1}{4\pi\epsilon} \sum_{i=1}^{N} \frac{q_i}{|\mathbf{R} - \mathbf{R}_i|}$$

$$V = \frac{1}{4\pi\epsilon} \int_{\mathcal{V}'} \frac{\rho_v}{R'}\, d\mathcal{V}' \quad \textbf{(volume distribution)},$$

$$V = \frac{1}{4\pi\epsilon} \int_{S'} \frac{\rho_s}{R'}\, ds' \quad \textbf{(surface distribution)},$$

$$V = \frac{1}{4\pi\epsilon} \int_{l'} \frac{\rho_\ell}{R'}\, dl' \quad \textbf{(line distribution)}.$$

Electric Field as a Function of Electric Potential:

We know how to expressed V in terms of a line integral over E.

$$dV = -\mathbf{E} \cdot d\mathbf{l}.$$

$$\int_{P_1}^{P_2} dV = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l},$$

For a scalar function V, the above equation rewrite as

$$dV = \nabla V \cdot d\mathbf{l},$$

We can compare above two equations, getting

$$\mathbf{E} = -\nabla V$$

"This differential relationship between V and E allows us to determine E for any charge distribution by first calculating V and then taking the negative gradient of V to find E".

Poisson’s Equation:

With D = εE, the differential form of Gauss’s law given by

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon}$$

$$\nabla \cdot (\nabla V) = -\frac{\rho_v}{\epsilon}.$$

By using Laplacian of a scalar function V,

$$\nabla^2 V = \nabla \cdot (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon} \qquad \textbf{(Poisson's equation)}.$$

Poisson’s and Laplace’s equations are useful for determining the electrostatic potential V in regions with boundaries on which V is known, such as the region between the plates of a capacitor with a specified voltage difference across it.

Thank you !