Modeling of Hybrid Dynamical Systems – An Approach to solve complex power electronics circuit

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Abstract

Technological innovation pushes towards the consideration of dynamical systems of mixed continuous and discrete nature, which are called "Hybrid systems". Hybrid systems arise, for instance from the combination of analog-continuous time process and digital time-asynchronous controller. Many a system like consumer products and physical system contain a hybrid nature. This paper presents first general approach for modeling of hybrid system and then we have presented a complete hybrid model of Buck and Boost converter.

Keywords: *Hybrid system, Hybrid control, Buck-Boost converter*

I. INTRODUCTION

Generally speaking, hybrid systems are mixtures of real-time (continuous) dynamics and discrete events. These continuous and discrete dynamics not only co-exist, but interact and changes occur both in responses to discrete instantaneous, events and in response to dynamics as described by differential or difference equation in time. One of the main difficulties of hybrid systems is that the term "hybrid" is not restrictive – the interpretation of the term could be stretched to include virtually any dynamical system we can think of.

From a general system-theoretic point of view one can look at hybrid systems as systems having two different types of ports through which they interact with their environment. One type of ports consists of the communication variable. The variable associated with these ports is symbolic in nature, and represent data flow. The second type of ports consists of the physical ports, where the term physical would be interpreted in broad sense. Thus a hybrid system can be regarded as a combination of discrete and continuous dynamics. The main problem in the definition and representation of a hybrid system is precisely to specify the interaction between this symbolic and continuous dynamics [1].

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Towards a definition of hybrid system

From a conceptual point of view the most basic definition of a hybrid system is to immediately specify it's our goal is to develop a mathematical model of hybrid systems rich enough to describe both the evolution of continuous dynamics and the discrete switching logic, and capable of modeling uncertainty in both the continuous and discrete input variables. In this section, we present a hybrid system model that was developed in [2] and is based on overlaying finite automata on nonlinear continuous-time control systems. To get the ideas fixed, we start with finite-state automata and continuous state, continuous time control systems.

Definition (Continuous-time state-space model): A continuous –time state-space is described by a set of state variable x taking values in R^n and set of external variables w is taking values in R^q related to mixed set of differential and algebraic equations of the form

$$F(x, \dot{x}, w) = 0$$

Of course, the above definition encompasses the more common definition of a continuous-time input-state-output system

$$\dot{x} = f(x,u)$$
$$y = h(x,u)$$

Definition (Finite automaton): A finite automaton is described by a triple *(L, A, E),* here L is a finite set called the state space. A is finite set called the alphabet whose elements are called symbols, E is the transition rule, it is called subset of $L \times A \times L$ *and its elements* are called edges.

II. MODELING OF HYBRID SYSTEM

Our goal is to develop a mathematical model of hybrid systems rich enough to describe both the evolution of continuous dynamics and the discrete switching logic, and capable of modeling uncertainty in both the continuous and discrete input variables. In this section, we present a hybrid system model that was developed and is based on overlaying finite automata on nonlinear continuous-time control systems. To get the ideas fixed, we start with finite-state automata and continuous state, continuous time control systems.

A. Notation: Let *W* be a countable collection of variables and let W denote its set of valuations, that is, the set of all possible assignments of the variables in W. We refer to

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variables whose set of valuations is countable as *discrete* and to variables whose set of valuations is a subset of a Euclidean space R^n as *continuous*. We assume that Euclidean space is given the Euclidean metric topology; whereas countable and finite sets are given the discrete topology (all subsets are open). Subsets of a topological space are given the subset topology and products of topological spaces are given the product topology. For a subset U of a topological space we use to $\overline{U}$ denote its closure, U^0 its interior, ∂u its boundary, U^c its complement its cardinality, 2^u the set of all subsets of U , U^w the set of finite or infinite sequences of elements in U , and the set of piecewise continuous functions R from to U [2] .

A finite-state automaton is represented as:

$$\left(Q, \Sigma, Init, R\right) \quad (1)$$

Where Q is a finite set of discrete state variable: $\Sigma = \Sigma_1 \cup \Sigma_2$ is a finite set of discrete input variable, where Σ_1 contains the controller's input and Σ_2 contains environment's input, which cannot be controlled: $Init \subseteq Q$ is a set of initial states: and $R : Q \times \Sigma \to 2^q$ maps the state and input space to subset of state-space and thus describes the transition logic of the finite automaton. An execution of (1) is defined to be finite or infinite sequence of states and inputs $\left(q(\cdot), \sigma(\cdot)\right) \in Q^w \times \Sigma^w$. $i \in z$ $q(0) \in Init$ And $q(i+1) \in R(q(i), \sigma(i))$.

Continuous-time state-space control system on the other hand may be represented as a differential equation evolving on X as

$$\dot{x} = f(x, v) \quad (2)$$

Where $x \in X$ is the state where X = R^n: $v \in V = U \times D$, is the space of continuous input variable, $U = R^u$ is the set of control inputs and $D = R^d$ is the set of disturbance inputs: f is a vector field assumed to be a globally Lipschitz on x and continuous in v and the initial state $x(0) \in Init$ where $Init \subseteq X$. A trajectory of (2) over an interval $[\tau, \tau'] \subseteq R$ is map $\left(x(\cdot), v(\cdot)\right) : [\tau, \tau'] \to X \times V$ such that $\dot{x} = f(x(t), v(t))$ for all $t \in [\tau, \tau']$.

Hybrid Automaton: Since we are interested in hybrid phenomena that involve both continuous and discrete dynamics, we introduce the hybrid time trajectory, which will encode the set of times over which the system is defined.

A hybrid automaton is a collection

$$H = (Q, X, \Sigma, V, Init, f, Inv, R)$$

Where

- $Q \cup X$ is a finite collection of state variable with Q finite and X = R^n
- $\Sigma = \Sigma_1 \cup \Sigma_2$ is finite collection of discrete input variable, where Σ_1 is the set of discrete control input and Σ_2 is set of discrete disturbance input.
- $V = U \cup D$ is the set of continuous input variables, where U is the set of continuous control inputs and D is the set of continuous disturbance inputs.
- $Init \subseteq Q \times X$ is a set of initial states
- $f : Q \times X \times V \to X$ is vector field
- $Inv \subseteq Q \times X \times \Sigma \times V$ is called an invariants and defines combination of states and inputs for which continuous evolution is allowed.
- $R \subseteq Q \times X \times \Sigma \times V \to 2^{Q \times X}$ is reset relation which encodes the discrete transition of the hybrid automaton.

We refer to $(q, x) \in Q \times X$ as the state of Hand $(\sigma, v) \in \Sigma \times V$ as the input of H. We make the following assumption to ensure that the hybrid automaton does not *block* trajectories, causing the system to deadlock: assume that Inv is open set that of $(q, x, \sigma, v) \notin Inv$ and then $R(q, x, \sigma, v) \neq \phi$.

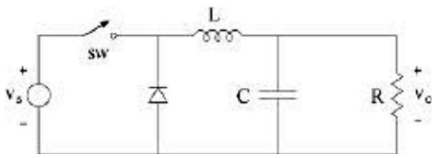
The main differences between the model presented here and that of timed and linear hybrid automata are in the continuous dynamics: we incorporate full nonlinear models of the continuous state dynamics and include continuous input variables to model both parameters that the designer may control and disturbance parameters that the designer must control against. This allows an accurate representation of the continuous physical processes that we would like to model and control.

Now we are going to model the buck and boost converter hybrid modeling here. First we understand the circuit equation then we convert it into state space form and then we prepare hybrid model. For the sake of easiness we present here only buck converter hybrid model because there is no difference between modeling of the converter except vector field F equation.

In the circuits we will suppose switch sw is controlled by a binary periodic signal *clock*(*t*), with period T_s, being $t_{on} and t_{off}$ the times during which the function value is 1 and 0 respectively. We will consider three states, S_{on}, S_{off} and S_{nc}. The states S_{on} and S_{off} will be associate to operation modes with switch sw in states *on* and *off*, respectively, whereas the state S_{nc} will be associate to the mode in which the diode does not conduct (null intensity). In this way we are going to analyze the both *buck* and *boost* converters. We will denote by $I_l(t)$ the intensity through the coil and $V_c(t)$ the voltage across the capacitor.

III. BUCK CONVERTER

The converter will give us output voltage smaller than the input supply voltage. The circuit is given below [3]:



If sw is closed, then diode is in reverse mode and can be removed for analysis. The electrical equation can be written in the form given below:

IV. $$\frac{dI_l(t)}{dt} = \frac{-1}{L}V_c(t) + \frac{1}{L}V_s(t)$$

V. $$\frac{dV_c(t)}{dt} = \frac{1}{C}I_l(t) - \frac{1}{RC}V_c(t)$$

When opening the switch sw, whenever the current is $I_l(t)$ is positive, it will also flow through the diode and the remaining circuit can be written as follows:

VI. $$\frac{dI_l(t)}{dt} = \frac{-1}{L}V_c(t)$$

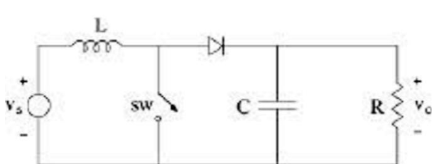
VII. $$\frac{dV_c(t)}{dt} = \frac{1}{C}I_l(t) - \frac{1}{RC}V_c(t)$$

In switch off mode with $I_l(t)$ >0, the voltage $V_c(t)$ is positive increasing and current will be decreasing and if t_{off} the circuit is long enough then will be given in equation form:

$$\frac{dI_l(t)}{dt} = 0$$

$$\frac{dV_c(t)}{dt} = -\frac{1}{RC}V_c(t)$$

IV. BOOST CONVERTER



In this circuit, the output voltage is higher than the input voltage. If sw is closed, then diode is in reverse mode and can be removed for analysis. The electrical equation can be written in the form given below:

$$\frac{dI_l(t)}{dt} = \frac{-1}{L}V_c(t)$$

$$\frac{dV_c(t)}{dt} = -\frac{1}{RC}V_c(t)$$

When opening the switch sw, whenever the current is $I_l(t)$ is positive, it will also flow through the diode and the remaining circuit can be written as follows:

$$\frac{dI_l(t)}{dt} = \frac{-1}{L}V_c(t) + \frac{1}{L}V_s(t)$$
$$\frac{dV_c(t)}{dt} = \frac{1}{C}I_l(t) - \frac{1}{RC}V_c(t)$$

In switch off mode with $I_l(t)$ >0, the voltage $V_c(t)$ is positive increasing and current will be decreasing and if t_{off} the circuit is long enough then will be given in equation form:

$$\frac{dI_l(t)}{dt} = 0$$
$$\frac{dV_c(t)}{dt} = -\frac{1}{RC}V_c(t)$$

State Models

As we use the state-space model for the linear system, here we use $I_l(t) = x_1$, $V_c(t) = x_2$, and $V_s = u$ and the state-space model is now

$$\dot{x} = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

and the matrices for each of the mode in converters are:

$$A_1 = \begin{pmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{pmatrix}, \; B_1 = \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}$$
$$A_2 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{pmatrix}, \; B_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

V. HYBRID MODELS

Buck hybrid model

We are liked to specify the hybrid model by

$$H = (Q, E, D, F, G, R)$$

As we know that there are three states $1 = S_{on}$, $2 = S_{off}$ and $3 = S_{nc}$ and so $Q = \{1.2.3\}$.

The collection of the edges is given by

$E = \{(1,2),(2,1),(2,3),(3,1)\}$ originally we have two state variables $x_1 = I_l$ and $x_2 = V_c$ for sake of simplicity we took another $\dot{x}_3(t) = 1$ whose solution with $x_3(0) = 0$ and so we have n=3.

The collection of domains is

$D = \{D_i := \{i\} \times R_{\pm}{}^{3}, i \in Q\}$, the vector field of the system is $F(t) = \begin{pmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{pmatrix}$

And $F_i(t) = \begin{pmatrix} A_{ij} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{pmatrix} + \begin{pmatrix} B_{ij} \\ 0 \end{pmatrix} u(t)$.

The guards are

$$G(1,2) = \left\{ \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \in R^3; x_3 = t_{on} + T_s Z_+ \right\}$$

$$G(2,1) = \left\{ \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \in R^3; x_3 = T_s Z_+ \right\}$$

$$G(2,3) = \left\{ \begin{pmatrix} X_1 & X_2 & X_3 \end{pmatrix} \in R^3; x_1 = 0 \right\}$$

$$G(3,1) = \left\{ \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \in R^3; x_3 = T_s Z_+ \right\}$$

Finally the collection of reset is

$$R = \left\{ R_c(x) = i_{d(x)}; x \in G_e \right\}$$

VI. CONCLUSION

It is shown hereby that with the use of hybrid dynamical systems, we can easily solve electronic circuit problems. Here we are using Buck Boost Converter for the solution of the problem.

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